

# **Realignment of Debt Service Obligations and Ability to Pay in Concessional Lending: Feasibility and Modalities**

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Comments are welcome.

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## 1. Introduction

We consider schemes which have the potential to increase the flexibility of heavily indebted primary producing countries in meeting their debt service obligations. The paper is technical and confined to the feasibility and modalities of such schemes. It is not the purpose of this paper to advocate these schemes but rather to explore the best means of implementing a scheme of this sort conditional on a decision to proceed.

The structure of the paper is as follows. In section 2, we discuss the objectives of possible and their basic operating principles. Sections 3 discusses various modalities for achieving these objectives. In section 4, we discuss choice of the quantitative hedge parameters. Then, section 5 discusses simulation results for six severely indebted countries. Section 6 examines the performance of the scheme in three countries – Benin, Burundi and Ghana – in the first of which it operates reasonably well and in the second less well. Section 7 concludes and indicates directions in which further work is required.

## 2. Objectives and Principles

Indebted countries may benefit significantly if their debt service obligations can be matched more closely with either their export revenues or their export prices. In this section, we set out some of the issues of principle that should guide construction of and choice between such schemes.

### The Overall Objective

Concessional debt service typically shows relatively little cyclical variation over time. Export earnings are highly variable and, because commodity prices are at least to some extent mean reverting, this variation is in part cyclical. The main objective of the scheme we investigate is to more closely match the debt service obligations facing indebted governments with their ability to meet these obligations. Because schemes to smooth export earnings have not, in general, proved either successful or financially viable, this involves unsmoothing concessional debt service.

It is necessary to specify the objective more precisely. Alternative aims might be

- reduction in the variability of country's debt-export ratios; or
- reduction in the variability of the foreign exchange resources that a country has available after meeting debt obligations.

The debt sustainability literature has typically operated in terms of the former concept which has the merit of direct comparability across countries. However, for any particular country, the latter measure gives a more immediate measure of the impact of debt service payments.

We have posed these objectives in terms of variabilities, but it is arguable that what is important is avoidance of bad outcomes, whether in the form of high debt service to export ratios or low residual foreign exchange variability. In practice, these two

objectives are likely to be similar if it is also required that reduced debt service in poor times is matched by increased debt service in good times. The former, variance reduction, objective is simpler to formalize, so we will work with this in what follows. However, we will generalize the normal comparison of debt service payments and export earnings objective to take into account oil imports, which we will treat symmetrically with commodity exports.

Historically, indebted countries have typically borrowed from a range of official and non-official lenders. Some official loans are completely concessional, others are non-concessional and the majority are part concessional and part non-concessional. This latter group can be split into concessional and non-concessional components. So long as we focus on highly indebted countries, it is reasonable to suppose that, in the future, debt service will come to predominantly reflect concessional lending as non-official lending has ceased to be available to these countries. For this reason, and because our concern is to devise a scheme which may be implemented in a simple and straightforward way, we consider only concessional lending.

#### Insurance or Increased Concessionality

A fundamental issue is whether the proposed arrangements are to be seen as

- Modifying to the contractual obligations of borrowing governments in new, and perhaps also existing, concessional loan agreements: Donor governments might commit to either cancelling or postponing debt service requirements in years in which a price or export revenue shortfall is signalled as having occurred. The direct incidence of this arrangement would fall on creditor governments. This concession might be justified on the basis that creditor countries are typically importers of the same commodities that debtor countries export, and that low export prices or revenues are therefore a benefit to the creditor countries, although this benefit will often be too thinly spread across their citizens to be easily discerned.
- Providing a mechanism which will facilitate governments in meeting existing and unchanged contractual obligations. The scheme would operate through postponement of debt service payments in periods in which export prices or earnings are low, and bringing such payments forward in periods in which they are high. This the arrangement may be seen as a costless or near-costless means of increasing the flexibility and decreasing the burden of servicing concessional debt.

These conceptions are alternatives, and one is not necessarily superior to the other. In what follows, we confine attention to schemes based on the second philosophy but without taking a view as the possible desirability of enhancing the concessionality of developing country debt.

#### Conditionality

Should the benefits to indebted countries under the scheme should be conditional or unconditional? A complication associated with conditionality is that the process for

judging whether a particular government will qualify becomes time consuming, and benefits from the scheme are therefore less timely. This was a serious practical problem with the EU Stabex scheme where delays in making payments are widely held to have contributed to these payments becoming pro- and not counter-cyclical.

This issue is closely linked to that of whether the scheme is to be conceived as enhancing concessionality or as providing insurance through increased flexibility. Conditionality may well be appropriate in the former case, but fits less easily with the latter, insurance-based, conception. These considerations argue for unconditional benefits in the context of an insurance-based scheme, or at least for conditionality to be exceptional.

### Export Revenues or Commodity Prices?

Should the scheme relate to export revenues (compare the IMF Compensatory Finance Scheme) or to the price of the export commodities on world markets?

- A link to export revenues would more closely align the scheme with debtor countries' ability to pay. A price link will fail to insure borrowers against quantity shocks, and will give to "basis risk" if the country's export price moves differently from the world price.
- Against this, export revenues are only known with a one or two year lag while prices are observed immediately. Schemes, such as the EU's Stabex scheme, which have operated in terms of export revenues, have been slow to disburse. Basing the scheme on prices would increase timeliness.
- Basing the scheme on export revenues could also result in governments being rewarded for changes in the quantum of exports for which they may be directly or indirectly responsible. It may also give rise to incentives to manipulate reported trade statistics. These moral hazard considerations argue for a price rather than a revenue link.
- A price link introduces the possibility for the lenders of using market-based hedge transactions to offset their exposure.

These considerations appear to use to argue on balance for a price rather than an export revenue basis for the scheme. However, a definitive answer to this central question must also depend on the relative efficacy of revenue and price-based schemes. Our contribution to this debate is to look at what can be attained from price-based schemes.

### Imported Commodities

Developing countries which lack domestic oil resources may find that oil imports pre-empt a significant proportion of their export revenues. Other developing countries import much of their grain requirements. Both groups of countries are vulnerable to upward movements in these import prices. This suggests it may be sensible to consider extension of the scheme to cover important imported commodities.

This extension turns out to be relatively straightforward. The schemes we consider below treat exports and imports symmetrically.

### The Commodity Price Trend.

With only a very few exceptions, primary commodity prices have tended to fall relative to the prices of manufactured goods. This fall has been documented for over one century. However, the pace of this fall varies from commodity to commodity, and, even more markedly, from one decade to another. This negative trend is generally regarded as resulting from the effects of productivity-enhancing technical change in production and intermediation processes, which, in the case of manufactures, is partly reflected in higher product quality and specifications. It seems likely that most primary prices continuing to fall relative to manufactures prices, but there is no reliable means of knowing by how much and which commodities, if any, will be exceptions to this pattern.

Concessional loans generally have long maturities (20-40 years) with initial grace periods of 10 or more years. It is clearly not possible to forecast likely levels for any commodity price over this type of horizon. This implies that extreme caution should be exercised in making commitments in relation to the absolute level of any price over a horizon of this length. It would be considerably more straightforward and less risky to confine the objective to one of matching the cyclical pattern of debt service obligations to those of export prices or earnings. In such a scheme, countries are assisted in adjusting to short to medium term fluctuations in their export revenues, but need to make other provisions to adjust to longer term structural changes in prices. An alternative justification for this separation is that it is inappropriate to address long term adjustment problems through insurance-type mechanisms.

### Simplicity

If the scheme is intended to address only cyclical fluctuations in export earnings, it will require a procedure for disentangling trend and cycle. It is important that any such scheme should be easily understood. The majority of the decomposition techniques discussed in the statistical and econometric literature fail to accord with the simplicity criterion. Furthermore, many are model-specific in the sense that different experts could arrive at different results possibly resulting in different payment obligations. It therefore seems in many respects simpler to estimate underlying trends by use of a moving average of past prices. This is the method which was used in some international commodity agreements, most notably the International Natural Rubber Agreement, which ceased operations in 1999, in updating price support bands. While not without problems, a moving average procedure has the considerable merits of being unambiguous and easily understood.

### Inactivity

Should the scheme address normal or only exceptional movements of commodity prices? If the latter route is taken, the scheme would be inactive for much of the time in the sense that the contractual changes would leave debt service unaltered in periods in which

commodity export and import prices are at “normal” levels, if low export prices are offset by low import prices, or vice versa. There are two arguments for the inactive approach:

- There is some evidence, which we do not regard as conclusive, that large commodity price shocks have disproportionate effects on macroeconomic performance and growth. In particular, some take the view that while growth suffers in periods of low export prices, the revenues from high export prices are often squandered and do not generate sustained (or even any) increases in growth.
- Inactivity emphasizes the primacy of the original loan agreements.

We do not take a prior position on this issue but we report results below which suggests that an inactivity filter tends to reduce beneficial as well as non-beneficial interventions.

### Summary

We consider schemes aimed at reducing the variance of the ratio of, or the difference between, indebted countries’ debt service to their export earnings. The schemes will work in terms of providing increased flexibility in the timing of debt service payments through postponement and acceleration of payments, but will not involve any additional increase in concessionality. The additional flexibility offered under the scheme will not normally be associated with conditionalities. We argue that, on balance, the scheme should work in terms of world prices and not export earnings, since this increases timeliness and reduces moral hazard risk, albeit at the expense of offering borrowing countries a less than perfect hedge. The scheme can be generalized to cover imported commodities, such as oil. However, whether on the export or the import side, the objective will be limited to offering flexibility in timing in response to cyclical movements in commodity prices, as measured by deviations from a moving average trend, and not from adverse movements of the trend itself. There may be merit in designing the scheme such that it only comes into effect in periods of exceptional export or import prices.

## **3. Modalities**

### Alternative Instruments

The most simple way to consider modification of the repayment structure of existing concessional loans to give an element of flexibility in relation to commodity price movements is to combine the current loans with a set of floating-for-fixed commodity swaps or alternatively a set of floating-for-fixed commodity swaptions (ie options based on the floating for fixed swaps). This gives an overlay structure. Crucially, operating according to this modality limits the size of the implied commodity hedge to the debt service obligations at the time the overlay is negotiated. In line with this observation, we regard these debt service obligations as the instrument in terms of which the commodity price conditionality is provided. It is immediate that this type of instrument will be more effective the higher the country’s debt service to export ratio, since this will permit provision of larger positions.

There is a number of possible alternative instruments. Since we are primarily concerned with the relationship between export earnings and the ability to service concessional debt,

we use the anticipated level of export earnings as an alternative instrument. In this approach, the country is provided with a set of commodity swaps or swaptions which augment rather than overlay its debt service obligations. This is tantamount to provision of an export revenue hedging facility for indebted countries. As set out above, such an arrangement is independent of a country's debt service obligations, and this may be the first best approach. However, if such a scheme is to be proposed as a means to achieving debt sustainability, it will be necessary to relate the payments under the scheme to debt service obligations. In what follows, we cap the hedging benefit to a country such that, in times of adverse prices, the country's net debt plus hedge payment cannot become negative (ie the country must be a net payer), and that, in times of favorable prices, the net payment the country is obliged to make is capped at double the original debt service payment.<sup>1</sup>

### Commodity Swaps

Floating-for-fixed interest rate swaps, sometimes known as "plain vanilla swaps", are the most widely traded interest rate derivative instruments. High quality borrowers have a comparative, as well as an absolute, advantage in borrowing at fixed interest rates relative to lower quality borrowers. (This is because lenders will require the flexibility to raise rates against low quality borrowers if their positions further deteriorate). However, these borrowers will welcome the opportunity to swap out some of their floating interest rate exposure in favour of fixed rates, and high quality borrowers will be able to reduce their overall borrowing costs by agreeing to such swaps. Commodity swaps extend this principle from interest rates to commodity prices.

One way of thinking of the commodity swap proposal is to see it as an extension of the practice, which already exists for some multilateral lending, of structuring a loan in terms of a basket of currencies. The currency basket approach allows the borrower to match the currency composition of its debt service obligations to the currency exposure of its export revenues. This is formally equivalent to augmentation of the original loan with a set of currency swaps (euros or yen for dollars).

The principle of commodity swaps may be illustrated by reference to the gold loan market. Many fledgling gold mining companies finance mine developments by taking out gold loans. Under such arrangements, they borrow a specified quantity of gold from a broking house, but effectively from central banks which wish to mobilize gold reserves, and use the monetary value of the borrowed gold to finance their activities. When the loan matures, they either deliver newly mined gold to the broker (and hence back to the lending central bank), or more normally, repay the current monetary value of the gold loan principal. There will be a gold interest rate. The advantage of this arrangement for the mining company is that it combines a financing and a hedging transaction – provide the mine produces, it can repay its loans whatever the gold price. Participating central banks earn interest on otherwise unremunerative gold reserves. A gold loan may be considered as a normal currency loan plus a fixed-for-floating gold swap in which the mining company assumes a floating rate liability.

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<sup>1</sup> We are grateful to Mark Dorfman for suggesting this modification to the overlay scheme.

How does this apply to poor indebted countries? Most poor countries remain highly dependent on a small number of primary commodity exports, and indeed, this may be one reason why they remain poor. We may consider such countries as having an asset (their primary commodity export revenues) which is exposed to one or more floating commodity prices. Their liability structure is currently fixed, in the sense that debt service is independent of the commodity prices. They will benefit by swapping out this fixed rate exposure for a floating rate exposure which matches their floating commodity price exposure.

Note that the commodity swap structure is simply an organizing principle and does not have any necessary or even likely implication that contracts of this sort can be achieved through a set of market swaps. We return to this issue later. Irrespective of whether or not there are any market-based offsetting arrangements, there are three clear advantages of this formalization:

- it emphasizes the applicability of standard financial theory of hedging to the structuring of the instruments;
- it allows valuation of the instruments under standard assumptions; and
- it underlines that the proposed contractual modifications are natural extensions of instruments which already exist and are traded in the market place.

Acknowledging these advantages, it is nevertheless important that we should not allow these structures to straitjacket the proposals which we consider.

### Coverage

Should the scheme aim to provide a comprehensive coverage of a country's export and import commodities or confine itself just to the major export and import commodities? We have chosen just to focus on the most important exports and on oil imports because

- many less important commodity exports lack clearly defined international prices;
- the focus on a small number of major export and import commodities works towards a simple and easily understood structure;
- inclusion of less important export and import commodities will have a relatively small effect on outcomes.

### Swap Structure

A loan of \$ $A$  matures in period  $T$ . Scheduled repayments are  $S_1, S_2 \dots S_T$ . For IDA loans  $T = 40$  and

$$S_t = \begin{cases} 0 & 0 < t < 10 \\ 0.02A & 10 \leq t < 20 \\ 0.04A & 20 \leq t < 40 \end{cases} .$$

These payments are fixed. Consider a floating price  $P_t$  with initial level (at loan inception)  $P_0$ . A corresponding floating rate loan overlay would require payments of

$R_t = \left(\frac{P_t}{P_0}\right)S_t$  with the consequence that repayments would be higher than on the corresponding fixed loan when  $P$  is high and lower when low. More generally, suppose a fraction  $\lambda \geq 0$  of the original loan is swapped into the floating loan. Repayments are

$$R_t = \left[ \lambda \left( \frac{P_t}{P_0} \right) + (1 - \lambda) \right] S_t = \left[ 1 + \lambda \frac{P_t - P_0}{P_0} \right] S_t = (1 + \lambda p_t) S_t \quad (1)$$

where  $p_t = \frac{P_t - P_0}{P_0}$ . Equation (1) makes the swap interpretation of the structure explicit.

Alternatively, if the instrument is the anticipated level of exports  $\tilde{X}_t$ , the swap would

require payments of  $\lambda \left( \frac{P_t - P_0}{P_0} \right) \tilde{X}_t$ . Total debt service would be

$$R_t = \lambda \left( \frac{P_t - P_0}{P_0} \right) \tilde{X}_t + S_t = \left( 1 + \frac{\lambda}{\psi_t} p_t \right) S_t \quad (2)$$

where  $\psi_t = \frac{S_t}{\tilde{X}_t}$ , the ratio of the original level of debt service to anticipated exports. In

what follows, we measure anticipated exports from a moving average of past levels of exports. Under the cap-floor scheme we set out above to ensure that modified debt service does not depart over far from the original debt service levels, equation (2) becomes

$$R_t = \begin{cases} 2S_t & \text{if } \lambda p_t > \psi_t \\ \left( 1 + \frac{\lambda}{\psi_t} p_t \right) S_t & \text{otherwise} \\ 0 & \text{if } \lambda p_t < -\psi_t \end{cases} \quad (3)$$

### Commodity Swaptions

Commodity swaption structures are a variant of the commodity swap structure considered above. They go some way towards meeting the principle that the scheme should aim to cope only with exceptional price movements.

A swaption is an option on a swap. A floating-for-fixed swaption may either be a call (the right to swap into floating from fixed) or a put (the right to swap into fixed from floating). We shall consider collar-type structures in which the borrowing country is long a set of out-of-the-money calls and short an equal number of out-of-the-money puts. because both set of options are out-of-the-money, debt service payments will be unaffected for moderate variations in the commodity price.

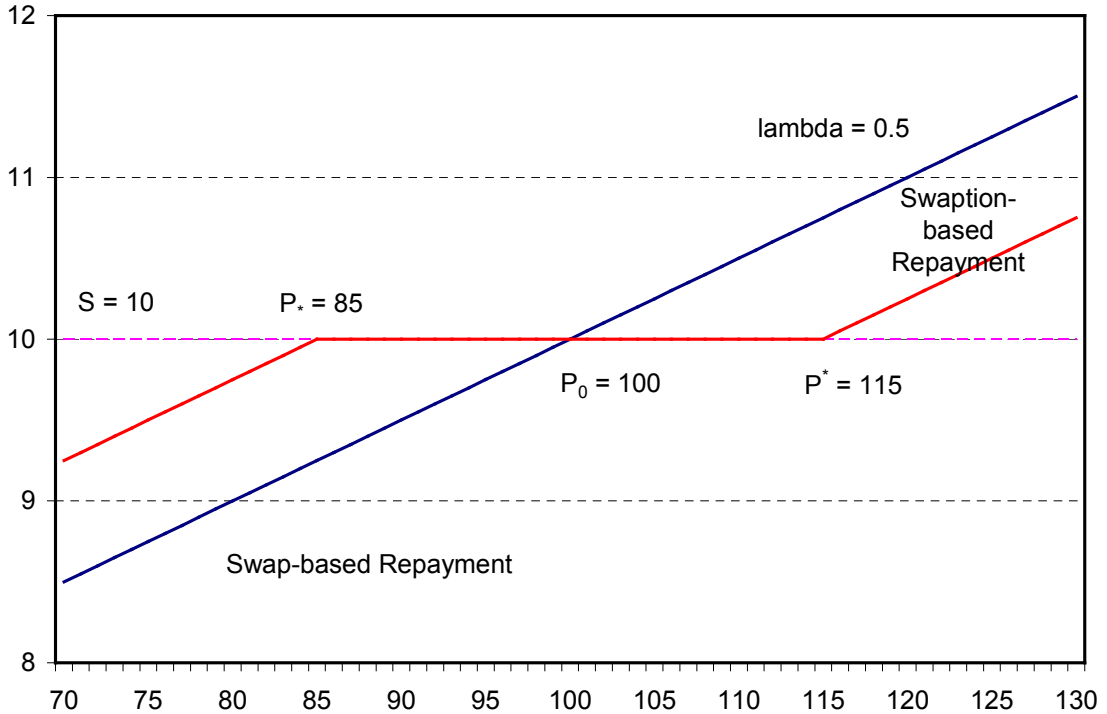
The swaption repayment scheme corresponding to the swap scheme in equation (1) may be expressed as

$$R_t = \begin{cases} \left[ 1 + \lambda \frac{P_t - (1 + \theta)P_0}{P_0} \right] S_t & \text{if } (1 + \theta)P_0 < P_t \\ S_t & \text{if } (1 - \theta)P_0 \leq P_t \leq (1 + \theta)P_0 \\ \left[ 1 - \lambda \frac{(1 - \theta)P_0 - P_t}{P_0} \right] S_t & \text{if } P_t < (1 - \theta)P_0 \end{cases} \quad (4)$$

The middle row of equation (4) shows that in the band in which the commodity price is within  $\pm\theta\%$  of  $P_0$ , debt service payments are unchanged from those under the standard currency loan. If the price falls short of  $(1 - \theta)P_0$ , debt service is reduced in relation to the proportion  $\alpha$  of this shortfall. Correspondingly, at prices above  $(1 + \theta)P_0$ , debt service is increased in relation to the proportion  $\alpha$  of the excess. This may be expressed more succinctly as

$$R_t = \left[ 1 + \lambda \max(p_t - \theta, 0) + \lambda \min(p_t + \theta, 0) \right] S_t \quad (5)$$

Equations (4) and (5) makes explicit that the original loan for  $\$A$  has been modified by adding a short position of calls on  $\$A$  in the floating rate loan with strike price  $(1 + \theta)P_0$  and a long position of puts on  $\$A$  in the floating rate loan with strike price  $(1 - \theta)P_0$ .



**Figure 1: Swap-based and Swaption-based Price Conditionality**

These repayments are illustrated for a simple example ( $P_0=100$ ,  $P^*=(1-\theta)P_0=85$ ,  $P^*=(1+\theta)P_0=115$ ,  $\lambda=1/2$ ) in Figure 1, which also illustrates a simple swap-based scheme

Commodity swaption-based schemes are reminiscent of band stabilization schemes operated in fixed exchange rate currency regimes and of the buffer stock commodity stabilization schemes operated in tin (until 1985) and natural rubber (until 1999). These structures may be extended to use export revenues as the instrument, but once the caps and floors are taken into account, the algebraic expressions become too complicated to be very useful.

### Extension to Imported Commodities

The structures outlined above may be extended to include imported commodities, in particular petroleum. Countries importing these commodities may experience difficulties in meeting scheduled loan repayments in periods when imported commodity prices are high. This is achievable by allowing borrowing countries to make commodity-based loans to the lenders, or to take collar positions in such loans. These loans again have swap-based or swaption-based structure.

Write the oil import price as  $Q_t$  with initial (loan contract or inception date) price of  $Q_0$ .

Write  $q_t = \frac{Q_t - Q_0}{Q_0}$ . Then a swap-based scheme, generalizing that defined by equation

(1), is given by

$$\begin{aligned} R_t &= \left[ \lambda \left( \frac{P_t}{P_0} \right) - \mu \left( \frac{Q_t}{Q_0} \right) + (1 - \lambda + \mu) \right] S_t \\ &= \left[ 1 + \lambda \frac{P_t - P_0}{P_0} - \mu \frac{Q_t - Q_0}{Q_0} \right] S_t = (1 + \lambda p_t - \mu q_t) S_t \end{aligned} \quad (6)$$

One way of thinking about this structure is to see the original fixed repayment loan as increased by the fraction  $\mu\%$ , which is then lent back to the lender in the form of an oil-based or grain-based loan. The borrowing country receives fixed payments on  $\$ \mu A$  value of this component of the loan, but makes oil- or grains-linked payments to the lender. Similarly, the corresponding scheme in which exports are the instrument is governed by the equation

$$R_t = \begin{cases} 2S_t & \text{if } \lambda p_t - \mu q_t > \psi_t \\ \left[ 1 + \frac{1}{\psi_t} (\lambda p_t - \mu q_t) \right] S_t & \text{otherwise} \\ 0 & \text{if } \lambda p_t - \mu q_t < -\psi_t \end{cases} \quad (7)$$

The extension of equation (6) to the swaption-based scheme is defined by

$$R_t = \left[ 1 + \lambda \max(p_t - \theta, 0) + \lambda \min(p_t + \theta, 0) - \mu \max(q_t - \theta, 0) - \mu \min(q_t + \theta, 0) \right] S_t \quad (8)$$

where we have defined the bands symmetrically for import and export prices. The borrowing country is long the call and short the put in the export price but short the call and long the put in the import price.

The poor quality of oil import causes major problems for the extension of the scheme to imported oil. We comment on these problems below in section 6 with specific reference to Benin and Ghana. These information problems make it difficult to estimate appropriate hedge ratios for oil imports may also lead to underestimation of the beneficial effects of the schemes we investigate.<sup>2</sup>

### Variable Strike Swaps and Swaptions

The swap and swaption schemes considered above would give borrowing countries the benefit of repayments based on floating commodity prices for the entire lifetime, perhaps up to 40 years, of a concessional loan. If, as we anticipate is likely, commodity prices continue to decline, this would be a very considerable benefit, but one achieved at the expense of the lending countries. But it would also impose upon them the risk that scarcity might eventually strike reversing the downward trend of the past century. These risks will probably not be countenanced, certainly by lenders, but possibly also not by borrowers. In any case, as we noted in section 1, risk sharing instruments of this sort are not the most appropriate for coping with long term adjustment problems. For these reasons, we modify the instrument structures so that, in technical terms, they have time-varying strike prices.

We may think of the initial price  $P_0$  in the above schemes as being the contract strike price. In this sense, these are fixed strike instruments. Generalization to a floating strike is straightforward, simply by replacing  $P_0$  and  $Q_0$  in equations (1-3) by time-subscripted prices  $P_t^{av}$  and  $Q_t^{av}$ . This notation reflects the judgment that  $P_t^{av}$  and  $Q_t^{av}$  will be moving averages of past prices.

This generates a 2x2 matrix of possibilities:

		<i>Contract type</i>	
		<i>Swap</i>	<i>Swaption</i>
<i>Strike type</i>	<i>Fixed strike</i>	Floating for fixed swaps	Floating for fixed swaptions
	<i>Variable strike</i>	Floating for moving average swaps	Floating for moving average swaptions

The upper row defines the schemes considered in equations (1-8) and Figure 1. In what follows, we confine attention to schemes defined by the lower row of the table.

<sup>2</sup> Information is even less complete for the value of oil exports. Cameroon is the only country we consider which has significant oil exports. Valuation of reported tonnages of oil exported from Cameroon at world prices gives rise to figures which in many years are inconsistent with the reported dollar value of total exports.

### Activity Thresholds

One method of attaining the objective that the scheme only operates in periods in which prices are at exceptional (low or high) levels is to use the commodity swaption structure discussed above. This has the required effect if the country scheme involves only a single export or import commodity. In cases of multiple commodity exports or imports, and in those in which the scheme involves both export and import commodities, the swaption structure fails to allow for offsetting. In the case which arises most frequently, in which the country has one major commodity export (say coffee) and one major import (typically oil), one would wish to allow for the possibility that a low coffee export price may be offset by a low petroleum import price, leaving the country's terms of trade broadly unaffected.

One possibility would be to operate the scheme in terms of swaptions defined in terms of a country-specific terms of trade index. We have not explored this route for the following reasons:

- Use of a terms of trade index imposes equal weighting on commodity exports and imports. Developing country imports typically exhibit much lower degrees of concentration than their exports – few countries specialize on the import side.
- Even if an export and an import share were equal, this would not necessarily imply equal hedge ratios for the two products – see section 4. Definition of the scheme in terms of an index prejudices some of the hedging issues.
- Lending institutions are likely to find that it is more difficult to offset positions defined in terms of an index than those relating to individual export and import commodities.

For these reasons, we prefer a simpler method for restricting the impacts of the scheme. We suggest adoption of a threshold of say  $\pm\delta\%$  such that if modified payments under the scheme would differ from those under the original currency contract by less than  $\delta\%$ , the original contractual payments would stand. This proposed modification is simple and has the desired effect of limiting impacts, but does detract from the simple commodity swap or swaption interpretation of the scheme.

### Additional Practical Issues

Here we note three practical issues which will need to be addressed.

- Concessional lenders will probably wish to put a zero floor on debt serviced in any particular year. Lenders would probably wish to find some other way to address the problem of export prices becoming so low (or import prices so high) that a scheme would generate a net payment from the lender to the borrower. We incorporate a zero floor in the simulation results we report in section 4.
- The use of a moving average estimate of the underlying trend price is intended to ensure that, on average, interventions triggered by the scheme are as likely to be positive, entailing accelerated repayment of debt, as negative, involving postponement. In any particular case, however, payments will be higher or lower than under current currency-based loans, perhaps markedly so. If the principle is that these scheme aims only to postpone or accelerate debt repayment, it will be

necessary to modify the swap or swaption structure with provisions to ensure that the aggregate payment is not affected. This would require premature termination, in the case of accelerated payments, and extension, in the case of retarded payments.

- This last problem may be exacerbated by the negative drift in commodity prices which will have the result that a moving average trend estimate will tend to be a few percent too high. It would be possible to address this by deflating the moving average, perhaps by around 5%, but it is less clear that this would be an appropriate procedure for oil, the major import commodity for poor countries, where scarcity considerations may offset the price-reducing impact of productivity advances.

#### 4. Hedging

For simplicity, we focus on a country with a single major export, with export share  $\alpha$  and price  $P$ , and for which oil imports have price  $Q$  and notional share  $\gamma$  in total exports.<sup>3</sup> We also focus on a straightforward commodity swap structure, as defined by equation (4), with nominal value  $\$A$ . That structure swaps out  $\lambda A$  of the loan into a floating-for-fixed swap in terms of the price of the export commodity, and undertakes a reverse swap of  $\mu A$  into a floating-for-fixed swap defined in terms of the import price commodity. The borrowing country becomes long in exposure to the variable commodity export price but short in exposure to the variable commodity import price. The issue we analyze in this section is that of the appropriate values for the hedge parameters  $\lambda$  and  $\mu$ .

We approach this question as an application of standard financial markets theory of hedging. The starting point for the analysis is the share-based hedge which sets the export commodity swap parameter  $\lambda$  equal to the export share  $\alpha$  and the oil import swap parameter  $\mu$  equal to the “share”  $\gamma$  of oil imports in total exports. This choice is appropriate if the instrument is anticipated exports and cross correlations are unimportant – i.e. the commodity export prices does no affect other export or oil imports, and the oil price does not affect exports. It is straightforward to extend this to cases in which there is more than a single export commodity.

##### What is the Appropriate Hedge Ratio?

Hedging using the export and oil import shares effectively adopts a unit (dollar for dollar) hedge ratio. The most basic hedging theory teaches that hedge ratios will normally differ from unity. The variance minimizing hedge ratios,  $h$  and  $k$  for the export and import commodities respectively, will depend on the correlation between changes in the revenue stream to be hedged and changes in the price used to hedge this stream, and on the relative volatilities of the revenue stream and the hedge price. Furthermore, hedging

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<sup>3</sup> In most highly indebted, total imports considerably exceed total exports with the difference covered by grants, remittances and capital inflows. Expressing oil imports as a share of exports rather than imports implies equivalence between an extra dollar spent on oil and a dollar lost from a lower value of commodity exports.

using export shares considers the export and import side hedges independently. This is only valid if changes in the export and import commodity prices are mutually uncorrelated, if the changes in the export price are uncorrelated with changes in import expenditures and if changes in the import price are uncorrelated with changes in export revenues. These are strong requirements. Where these conditions do not hold, a portfolio approach is required which takes into account the covariance structure of the prices, export revenues and import expenditures. Once one takes these covariances into account, one obtains more complicated formulas for the hedge ratios which we denote by  $H$  and  $K$  – see appendix.

In practice, it is simpler to derive hedge ratios from multiple regression of the quantity whose variance one wishes to minimize on the relevant prices. We do this for quantity  $X - O - S$  which we interpret as residual forex availability. The precise regression takes

as dependent variable  $\frac{X_t - \tilde{X}_t}{\tilde{X}_t} - \frac{O_t - \tilde{O}_t}{\tilde{X}_t} - \frac{S_t - \tilde{S}_t}{\tilde{X}_t}$  where a tilde indicates a centered five

year moving average. This is regressed on the similarly defined export price deviation

$p_t = \frac{P_t - \tilde{P}_{t-4}}{\tilde{P}_{t-4}}$  and the oil price deviation  $q_t = \frac{Q_t - \tilde{Q}_{t-4}}{\tilde{Q}_{t-4}}$ , where, however, the price

moving averages are four year moving averages defined over the period  $t-6$  to  $t-3$ .

Backdating of the moving averages is required to ensure that the contracts are hedgable.

An advantage of the regression approach is that we can also apply it to calculate hedge ratios for different minimization objectives where exact formulas are unavailable. In particular, we are interested in looking at the variance of the debt service to export ratio

$\frac{S}{X}$ . We modify this ratio by including oil import expenditures in the numerator, as

competitive with debt service. Consider a 1% decline in revenue for a commodity export which reduces total export expenditure  $X$  by  $\alpha\%$ . This decline will be offset by a decline in debt service by  $\alpha\%$ . On the other hand, a 1% rise in expenditure oil import expenditure  $O$  will require a 1% offsetting decline in debt service. To ensure symmetry, we therefore scale oil import expenditure by its share of total exports,  $\gamma$ . The appropriate ratio to

consider is therefore  $s = \frac{S + \gamma O}{X}$ . We obtain hedge ratios by regressing deviations from

the centered moving average trend of  $s$ ,  $s_t - \tilde{s}_t$ , on the prices  $p_t$  and  $q_t$  defined above.<sup>4</sup>

Regression-based weights should generate a superior hedge to the share-based weights in the presence of correlations between commodity prices and oil prices and if these prices influence other expenditure and receipt items. The practical problem is whether it is possible to rely on these weights being constant over time. There will always be a worry that particular hedge ratios estimated over short samples of often poor quality data can generate hedges that may be less good than those obtained by use of simple export and

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<sup>4</sup> Regressions were performed over the period 1985-2009 (1999 for Cameroon because of lack of data).

import shares. The apparent superiority of the regression-based weights may therefore be illusory. We therefore report results based on both share and regression-weights. The former may be thought of as providing a lower bound and the latter an upper bound on attainable performance.

We estimate variance minimizing hedge weights for the swap-based schemes but not for the swaption or the reduced activity schemes. The nonlinearities associated with these two schemes implies that optimization would need to be within a Monte Carlo simulation framework. We do not pursue that direction in this paper.

### Welfare Analysis

We need a criterion by which to judge the relative success of different schemes. This is particularly important because of the differences which are likely to emerge between schemes which aim to minimize the variance of free forex and those which look at the debt-service to export ratio (as modified to take oil imports into account).

We propose to ask what reduction in debt service would leave countries indifferent between the reduced but otherwise unmodified debt service payments and debt payments under the proposed schemes but without any overall reduction. This provides a common metric in which alternative schemes can be evaluated both within and across countries. The measure depends on change in the variability of free foreign exchange (forex,  $X - O - S$ ) – see appendix for details.<sup>5</sup> It may be expressed either in percentage terms or as a dollar reduction.

The disadvantage of this approach is that it necessarily depends on an assumed risk aversion coefficient. If countries are highly risk averse, they will be willing to pay a lot for even a small degree of risk reduction, while if they have low risk aversion they would prefer even a small reduction in overall debt service to a large reduction in uncertainty. In what follows, we use a conservative value of unity for the partial risk aversion coefficient – see appendix.<sup>6</sup> Use of a higher value for risk aversion, perhaps 2.5, would increase the valuation of the risk reduction benefit proportionately.

### Summary

The choice of hedge parameters raises both technical and substantive issues. It will be necessary to exercise judgment and formulaic approaches can at best guide these decisions. On the technical side, the issue is one of the reliability and structural constancy of the covariance estimates that enter into the full formulas for calculation of the variance

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<sup>5</sup> The measure ignores any net transfers which might take place on the basis that these are required to sum to zero in dollar, if not in NPV, terms over the lifetime of the loans.

<sup>6</sup> Partial risk aversion is defined in relation to component of income, here exports, rather than income itself in the standard relative risk aversion measure – see appendix. We prefer this measure since use of a common relative risk aversion would entail a diversification effect whereby countries with lower export to GDP ratios would put a lower value on any reduction in uncertainty than would countries with a high export to GDP ratio.

minimizing hedge. To the extent that these estimates are regarded as problematic, one may be better advised to remain with simpler export and import share hedge parameters. We evaluate the impact of alternative schemes in a money metric by asking what is the equivalent reduction in overall debt service in an otherwise unmodified debt service schedule.

## 5. Simulations

We have performed historical simulations, over the period 1984-2000, of schemes of the form outlined above for a set of ten severely and moderately indebted African countries.

The countries are

Benin	Ghana	Rwanda
Burkina Faso	Kenya	Tanzania
Burundi	Madagascar	
Cameroon	Malawi	

This group of countries was selected from the set of all countries classified by the World Bank as severely or moderately indebted and which in addition satisfied the following three criteria

- Service on concessional debt is a sufficiently large proportion of total debt service to allow the scheme to have a significant impact on overall debt service.
- The country has at least one export commodity making up 10% of total exports or one import commodity accounting for 10% of total imports.
- The country publishes statistics on total imports and exports, with at most a few gaps, which allow calculation of the required hedge ratios.

Application of these three criteria reduced an original candidate list of many more countries to the ten listed above.

### Scheme Design

Section 3 set out a range of alternative designs. We need to choose a more limited number of schemes for simulation. In what follows, we will report on three schemes:

- A commodity swap scheme, as defined by equation (6), with price deviations measured relative to a four year moving average of past prices lagged one year (ie the moving average of prices five, four, three and two years previously).
- A swaption scheme, as defined by equation (8) and relating to the same moving average with a  $\pm\theta\%$  band (strike prices  $\theta\%$  above and below the moving average price trend). We set  $\theta = 15\%$ .
- A swap scheme, as in A above, but with a  $\pm\delta\%$  inactivity range. We set  $\delta = 15\%$ .

We consider each of these schemes as implemented on the basis of both debt service and exports as instrument. In each case, we consider both share-based and regression-based hedge parameters.

### Simulation Results

Simulation results are tabulated by country in Appendix Tables A1-A4. Tables A1 and A2 take as objective the minimization of the variance of countries' residual forex whereas

Tables A3 and A4 are based on minimization of the debt service to export ratio, modified to include oil imports. In Tables A1 and A3 the instrument is the original level of debt service while Tables A2 and A4 report simulations in which the instrument is the lagged export revenue moving average. For each country and each hedging scheme, results are provided for each of the swap-based scheme (scheme A), the swaption scheme (scheme B) using a  $\pm 15\%$  threshold, and the restricted swap scheme (scheme C) with a 15% inactivity threshold.

The simulation results differ markedly across countries but are fairly consistent across alternative schemes in any given country. Of the ten countries considered, Burundi and Rwanda are seen as obtaining greatest benefit from the proposed scheme.<sup>7</sup> Estimated benefits are very low in Benin, Ghana and Kenya.

<b>Table 1</b>					
<b>Median Summary Statistics for Simulation Exercises</b>					
<b>Swaption Schemes (Scheme A)</b>					
Minimization Objective	Instrument	Share-based positions		Regression-based positions	
		Variance reduction	Value	Variance reduction	Value
$Var(X - O - S)$	Scheduled repayments $S$	14.8%	0.4%	15.7%	0.7%
$Var\left(\frac{S + \gamma O}{X}\right)$	Scheduled repayments $S$	27.9%	\$0.3m	0.4%	\$0.1m
$Var(X - O - S)$	Trend exports $\tilde{X}$	29.3%	1.0%	29.3%	2.2%
$Var\left(\frac{S + \gamma O}{X}\right)$	Trend exports $\tilde{X}$	-90.0%	\$0.5m	18.0%	\$1.0m
<p>The table summarizes the median results for the Scheme A simulations reported in from Appendix tables A1-A4. Row 1 of the table relates to Table A1, row 2 to Table A2 etc. In each case, the variance reductions relate to the specified objective function – results are comparable across rows 1 and 3 and 2 and 4, but not between odd and even numbered rows. Values are given as percentages of scheduled debt service and as dollar values using a common assumed unit value for the partial risk aversion coefficient. <math>X</math> is the total dollar value of export revenues, <math>O</math> is the dollar value of oil import expenditures, <math>S</math> is the dollar value of total concessional debt service and <math>\gamma</math> is the sample average ratio of <math>O</math> to <math>X</math>.</p>					

These differences across countries motivate an alternative focus on average benefits. Our preference is to focus on median benefits, interpretable as the benefit to the average

<sup>7</sup> Benefits are calculated over the ten year period 1991-2000. The simulation results for Rwanda exclude the civil war years 1994 and 1995. The results are much less favorable without this exclusion.

country, rather than on the (typically higher) sample average which is heavily influenced by the Burundian and Rwandan performance. Median benefits for the swap scheme (scheme A) are listed in Table 1.

- The schemes generally do give some reduction in the target variances – typically of the order of 15%-30%. The debt-service based hedge is as effective as the export-based hedge in reducing the variance of the augmented debt service to export ratio, but only around half as successful in reducing the variance of the free forex measure.
- Because our valuation metric is based on free forex, seen as financing discretionary imports, the higher variance reductions deriving use of anticipated export revenues as a hedging base approximately doubles scheme valuations. However, these remain small at around 1%-2% of debt service. A higher assumed risk aversion coefficient could raise these valuations to the 2½%-5% range. Even relatively large reductions in the variability of the augmented debt service to export ratio only translate into relatively small welfare improvements.
- Use of regression-based weights rather than export and import share weights generates a more marked variance reduction when the minimization objective is the variance of free forex than when the objective relates to the augmented debt service to export ratio. As noted, the problematic nature of this procedure relates to the structural constancy of the regression weights.

Overall, the most important conclusions that follow from the results summarized in Table 1 are the small extent in variability and hence the low size of the overall benefits derivable from the scheme, and the relative superiority of gearing the scheme to export revenues rather than scheduled debt service in terms of the dollar valuation of the interventions.<sup>8</sup>

The second interesting comparison is between the swaps scheme (Scheme A), the swaptions scheme (Scheme B) and the capped swaps scheme (Scheme C). In Table 2 we report this comparison for the schemes which aim to minimize the variance of free forex and which adopt hedge ratios derived from regressions. The relative ranking of the alternatives is the same for other choices and these can be inferred from the detailed results reported in appendix tables A1-A4. Recall however that both schemes B and C use 15% thresholds – we have not investigated alternative choices for the threshold parameters and it is possible that some of these could lead to different rankings.

The results reported in Table 2 show the swaps scheme, Scheme A, to be uniformly superior to the other two schemes in terms of both variance reduction and the value of the associated welfare change. These results confirm that the results reported in Table 1 are reliable in relation to the other design parameters and to overall valuation of the interventions. The comparison between the swaptions scheme (scheme B) and the capped swaps scheme (scheme C) is less clear, with the latter giving the greater median variance

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<sup>8</sup> This latter conclusion is a direct consequence of the definition of the utility function used to evaluate the benefits. The more substantial point is that it is quite difficult to think of a sensible utility specification in terms of the debt export ratio particularly if this is also to acknowledge the importance of oil imports.

reduction but the former the higher welfare valuation. The sample average comparisons in appendix Table A2 and A4 indicate a clear ranking of the swaptions scheme over the capped swaps scheme.

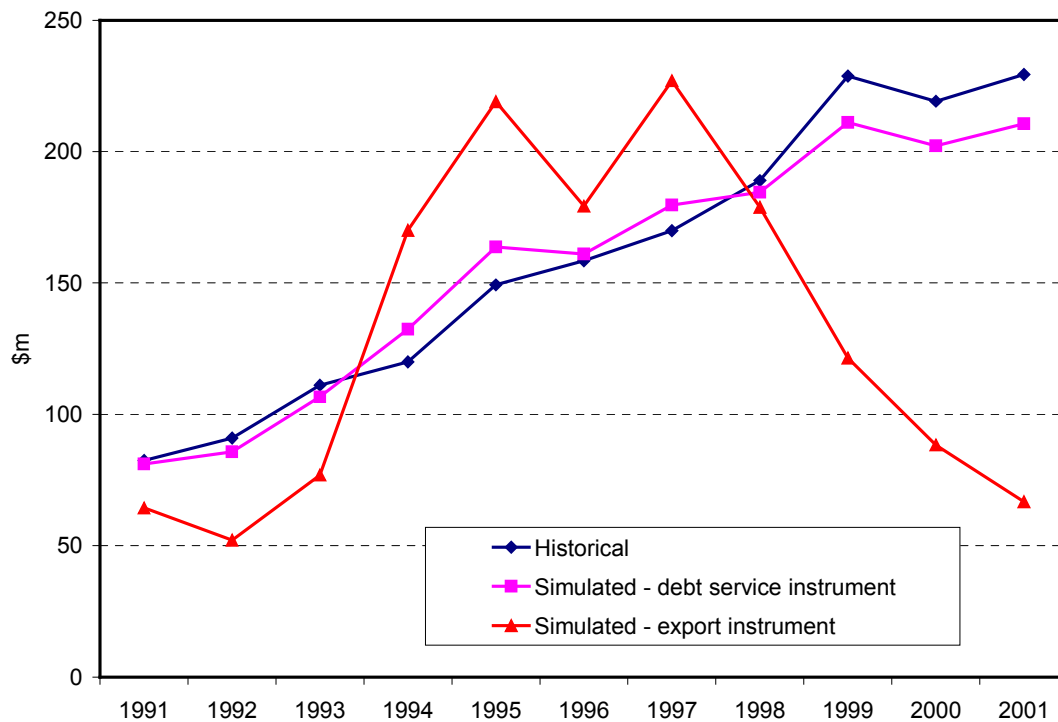
<b>Table 2</b>				
<b>Comparison of Alternative Schemes: Median Simulation Benefits</b>				
Objective: Minimization of $Var(X - O - S)$			Regression-based weights	
	Instrument	Scheme A	Scheme B	Scheme C
Variance reduction	Debt	14.8%	3.1%	12.9%
% benefit	service	0.7%	0.2%	0.2%
\$ benefit	$S$	\$0.3m	\$0.1m	\$0.1m
Variance reduction	Trend	29.3%	16.7%	26.7%
% benefit	exports	2.2%	1.1%	0.2%
\$ benefit	$\tilde{X}$	\$1.0m	\$0.7m	\$0.1m
<p>The table summarizes the median results for the Scheme A simulations reported in from Appendix Table A1 (rows 1-3) and Table A3 (rows 4-6). Values are given as percentages of scheduled debt service and as dollar values using a common assumed unit value for the partial risk aversion coefficient. <math>X</math> is the total dollar value of export revenues, <math>O</math> is the dollar value of oil import expenditures, and <math>S</math> is the dollar value of total concessional debt service</p>				

### Impact on IDA

Up to this point, we have considered only the potential benefits of the proposed schemes. They also have costs. The principal cost is the unsmoothing of flows from borrowing countries to the concessionary lenders. Here, we focus on the impact on IDA, the most important of these lenders. The impact is charted in Figure 2 which looks at Scheme A (swap based) schemes using regression weights using respectively the debt service and the anticipated (trend) export instruments. The figure compares the total debt repayments to IDA from the ten countries in our simulation sample with actual historical repayments over the simulation period 1991-2001.

There is a stark contrast between the impact of the scheme when the debt service instrument is adopted with that when the export instrument is used. Use of the debt service instrument results in relatively small net departures of debt repayments from their historical time path – the maximum divergence is \$19m (in 2001). By contrast, use of the export instrument generates much larger divergences - \$154m in 2000 and \$189m in 2001. The consequence is that, at the start of the current decade, IDA would be receiving one third of its current debt service payments from qualifying countries.

One reason for the large size of these figures is that there is relatively little offsetting – there is a large common factor in commodity price cycles. Net flows average 69% of gross flows in the scheme which uses debt service as instrument and 65% in the scheme which uses anticipated exports. The main element of offsetting in our sample comes from Cameroon whose oil export revenues are negatively correlated with commodity export earnings and oil import expenditures in the other countries in our sample.



**Figure 2: Historical and simulated repayments of IDA concessional debt, 1991-2001**

These results necessarily indicate a major question mark against the scheme using trend export revenues as the instrument. Table 1 suggests that the potential benefits from use of the export instrument are perhaps two to three times those from using the debt service instrument, but the disruption of IDA finances is many times larger than this. The choice is between a scheme which imposes a relatively small disruption on IDA finances but which generates a low expected level of benefits to participating countries, and an alternative which imposes much greater disruption on IDA for benefits which, although larger, remain low.

If a scheme along one of these two lines eventually comes to be proposed, it will be necessary to extend these calculations to NPV calculations, and perhaps also to look at IDA's VaR (Value at Risk) under the scheme. We have not done this, in part because such an exercise requires assumptions about how, and to what extent, IDA would offset its commodity risk exposure on the (exchange or OTC) financial markets. We believe such offsetting to be feasible, at least with regard to the principal commodities we consider.<sup>9</sup> However, offsetting will not be costless and offsetting costs must be expected to rise with the size of the position to be offset.

<sup>9</sup> There should be little problem with respect to coffee and cocoa. Tea and tobacco lack organized futures markets and would be more problematic. Cotton is in an intermediate position. Note that although concessional loans typically have long maturities, it would only be necessary to hedge at

## Summary and conclusions

The best performing scheme in our simulations was the swaps scheme based on anticipated (i.e. trend) export earnings and using regression-based weights. This gave benefits which we evaluate conservatively as equivalent to concessional debt service reductions of between zero (Benin) and 5¾% Rwanda). The median benefit is around 2¼% of concessional debt service. Use of a less conservative risk aversion parameter could double or perhaps even treble these estimated benefits, but that would be an upper bound. In particular, use of hedge ratios estimated from historic data is likely to over-estimate the extent of risk reduction attainable in the future.

Simulation of the resulting IDA flows indicates only a modest degree of offsetting across commodities. Tying the scheme to a debt service instrument implies relatively low disruption of payments to IDA. By contrast, use of trend exports as the hedging basis results in IDA assuming a high degree of commodity price risk with the potential for major disruption of its net repayment flows.

In summary, the simulation results are positive, in the sense that feasible adjustments of concessional debt service in relation to commodity export and import prices can improve the situation of highly indebted commodity-exporting countries, but negative, in that the estimated size of this benefit is relatively small.

## **6. Discussion of the Results**

In order to amplify the conclusions from the simulation exercises, we consider some general issues and then report on case studies for three of the ten countries for which we have conducted simulations: Burundi, Ghana and Benin. The discussion focuses throughout on the importance of quantity movements and of movements in the price basis (the difference between local and world prices).

### Basis Issues

Basis risk is the risk that the price which one is using to hedge certain transactions does not move perfectly with the price that one is attempting to hedge. Our concern in this context is that the prices countries obtain for their commodity exports may not be highly correlated with the international prices which we are using in order to construct the required commodity swaps. Table 3 reports these correlations.<sup>10</sup> The table shows the long term hedging basis to be generally good for coffee and cocoa and poor for cotton and tobacco. The correlations for tea are mixed.

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such time as the moving average price trends become revealed. IDA would therefore only need to hedge its positions a few years ahead, but would do this on a rolling basis.

<sup>10</sup> The table excludes gold and oil since for these two commodities we construct export and import values by multiplying quantities by world prices.

Country	Commodity	Correlation	Commodity	Correlation
Benin	Cotton	0.55		
Burkina Faso	Cotton	0.61		
Burundi	Coffee	0.87		
Cameroon	Coffee	0.93	Cocoa	0.91
Ghana	Cocoa	0.92		
Kenya	Coffee	0.85	Tea	0.92
Madagascar	Coffee	0.97		
Malawi	Tobacco	0.55		
Rwanda	Coffee	0.73	Tea	-0.26
Tanzania	Coffee	0.84	Cotton	0.60
	Tea	0.69		

The table gives the correlation of deviations of commodity export unit values from centered five year moving averages and the correspondingly defined deviations of the world prices from their moving averages. Sample: 1985-2001.

Coffee and cocoa are actively traded on international futures markets in London and New York. These prices are widely disseminated and serve the exporters who purchase in developing country markets. Cotton is traded in New York, but the New York price relates primarily to the protected US market. We take the *Cotton Outlook* (Liverpool A) price as benchmark, but this price is based on a survey of supposedly representative trades and is, in that sense, less transparent than a futures market price. It is also less widely disseminated. We use US import unit values as the world tobacco price. It is unsurprising that this is only modestly correlated with Malawian export unit values. Tea is more complicated because there is an active cash market in Mombasa, Kenya, and this price is well correlated with Kenyan unit values. The correlation with Tanzanian unit values is less good, and that with Rwandan unit values is actually negative.

In summary, basis risk does appear a significant problem for price-based schemes of the form we consider as soon one moves away from coffee and cocoa which appear to be priced relatively efficiently by international futures markets.

### Quantity Variation

Price-based schemes insure countries against movements in prices but not against quantity variations. We may analyze the extent to which this limitation is important by means of a simple decomposition. Using a different notation from the remainder of this paper, write the export revenue from a commodity as  $R$ , its price as  $P$  and the quantity exported as  $Q$ . We use lower case letters to denote the demeaned logarithms of the corresponding upper case variables. It follows that

$$r = \ln R - \overline{\ln R} = (\ln P - \overline{\ln P}) + (\ln Q - \overline{\ln Q}) = p + q$$

Denote  $\sigma_x^2 = Var(x)$  for any variable  $x$  and the covariance  $\sigma_{xy} = Cov(x, y)$  for any pair of variables  $x$  and  $y$ . Finally, write the quantity-price regression as  $q = \beta p + u$ . Since  $p$  and  $u$  are uncorrelated by construction, it follows that

$$\sigma_r^2 = (1 + \beta)^2 \sigma_p^2 + \sigma_u^2 = \left(1 + \frac{\sigma_{pq}}{\sigma_p^2}\right)^2 \sigma_p^2 + \sigma_u^2$$

The ratio  $\frac{\sigma_u^2}{\sigma_r^2} = 1 - \frac{\left(1 + \frac{\sigma_{pq}}{\sigma_p^2}\right)^2 \sigma_p^2}{\sigma_r^2}$  measures the extent of revenue variation that cannot be explained by contemporaneous price deviations and must therefore be attributed either to movements in the price basis or to uncorrelated quantity movements.

<b>Table 4</b>				
<b>Estimated Importance of Quantity and Basis Variation</b>				
<b>Country</b>	<b>Commodity</b>	<b>Correlation</b>	<b>Commodity</b>	<b>Correlation</b>
Benin	Cotton	80%		
Burkina Faso	Cotton	67%		
Burundi	Coffee	40%		
Cameroon	Coffee	44%	Cocoa	39%
	Oil	97%		
Ghana	Cocoa	78%	Gold	55%
Kenya	Coffee	64%	Tea	55%
Madagascar	Coffee	34%		
Malawi	Tobacco	59%		
Rwanda	Coffee	84%	Tea	76%
Tanzania	Coffee	51%	Cotton	88%
	Tea	21%		

The table gives the variance of deviations of log commodity revenues from centered five year moving averages which cannot be explained by deviations in international prices from centered five year moving averages. We interpret this measure as stating the importance of quantity and basis variations. Sample: 1985-2001.

Table 4 gives this ratio for the commodity exports we have considered in the simulation exercises. In general, prices explain less than 50% (and in some cases substantially less than 50%) of revenue movements. In our opinion, this is the major reason for the relatively weak performance of the schemes we have considered.

#### Country Study: Burundi

Burundi is the country for which the proposed schemes perform best, with the possible exception of Rwanda where the analysis is complicated by the civil war. Burundi's major export is coffee. This accounted for an average 79% of its total export revenues over our

simulation period of 1991-2000. This coffee was almost entirely of the arabica variety. The cost of oil imports averaged 31% of total export revenues over the same period.<sup>11</sup> Appendix Figure B1 graphs these receipts and expenditures. It is apparent that, in Burundi, coffee export revenue is highly variable while oil import expenditures are quite smooth. The major coffee revenue declines in 1993 and 1996 are arrowed for future reference.

Appendix Figure B2 shows Burundi's concessional debt service over the longer period 1984-2001. As bilateral concessional lending has declined, repayments of concessional debt have come to be dominated by IDA. The overall volume of concessional debt repayments has grown over time and Burundi's concessional debt service to export ratio has also increased to a 2001 level of 42%.

Appendix Figure B3 graphs the arabica (left hand axis) and petroleum prices (right hand axis) over the simulation period. Coffee prices were high in the mid-nineties but plummeted in 1999. At the same time, petroleum prices rose sharply from a previously relatively stable level. Table 3 emphasizes that Burundi generally experiences a very satisfactory hedging basis relative to the New York price for arabica coffee. However, movements in that price only explain around 40% of movements in coffee revenue – see Table 4. In that context, it is important to note that the coffee price rose slightly in 1993 from its depressed 1992 level, and in 1996 fell back moderately from a high 1995 level – see arrows.

Coffee export revenues were highly positively correlated with the world arabica coffee price ( $r = 0.57$ ) but oil import expenditures were weakly negatively correlated with the world oil price ( $r = -0.17$ ).<sup>12</sup> These correlations imply that any hedge should primarily relate to coffee and not to oil. The regression-based hedge ratios are 48% for coffee and 22% for oil, both less than their respective shares in total export revenues. The oil hedge parameter reflects a statistically significant negative correlation between the oil price and coffee exports ( $r = -0.48$ ) which implies, paradoxically, that although a short oil hedge will not protect oil import expenditures, it will hedge coffee export revenues.

Figure B4 compares Burundi's simulated concessional debt repayments under the export-based swap scheme A (using regression weights) compared with historic repayments. Payments are lower in five years, higher in three and broadly similar in three.<sup>13</sup> In particular, no repayments would have been made in the final two years of the sample (2000 and 2001) which saw the coincidence of low coffee and high oil prices. Figure B4 also graphs simulated debt service using debt service levels rather than exports as the instrument. Relative movements are similar to those generated by the export-based

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<sup>11</sup> Oil import expenditures, here and elsewhere, are inferred by multiplying reported quantities by 95% of the average Brent crude price.

<sup>12</sup> Here and elsewhere, reported correlations relate to the longer period 1985-2000 in order to give greater precision. All correlations relate to deviations from centered five year moving average trends.

<sup>13</sup> The simulations cover 1991-2001 (eleven years), but statistics are only calculated over the ten years to 2000 because of lack of full data for all countries.

scheme, but they are much smaller and the effects are lower. Finally, Figure B5 graphs the resulting quantity of free foreign exchange in the two simulated environments. The schemes are both somewhat successful in reducing the variability of this quantity, but neither does anything in relation to the sharp falls in coffee revenue in 1993 and 1996 which were not primarily related to price movements – recall that the coffee price rose slightly in 1993 and fell back modestly but at a high level in 1996.

Overall, the Burundi scheme operates reasonably well but fails to cope with two major revenue movements which were not (or were only slightly) determined by changes in world prices.

#### Country study: Ghana

The proposed schemes perform poorly in Ghana. Ghana has two major commodity exports – gold, which accounted for an average 40% of its total export revenues over our simulation period of 1991-2000,<sup>14</sup> and cocoa, which accounted for 25% of export revenues. The cost of oil imports averaged 24% of total export revenues over the same period. Appendix Figure G1 graphs these receipts and expenditures. Note that we estimated oil import figures by linear interpolation over the period 1993-95. The poor quality of oil import and export data is not unique to Ghana and is a major limitation on our ability to infer appropriate oil hedge ratios. Gold revenues fluctuate only mildly around a steady upward trend, while cocoa revenues and (so far as we can tell) oil import expenditures are more volatile.

Appendix Figure G2 shows Ghana's concessional debt service over the period 1984-2001. Bilateral concessional lending has been important in Ghana, but it has recently declined, and concessional debt repayments are now dominated by IDA. The overall volume of concessional debt repayments has grown over time but only in line with export revenues and Ghana's concessional debt service to export ratio has only increased modestly to a 2000 level of 10%.

Appendix Figure G3 graphs the cocoa and gold prices (left hand axis) and petroleum prices (right hand axis) over the simulation period. Both export prices were moderately favorable until the mid-nineties but gold fell from 1995 and cocoa, more sharply, in 1999. At the same time, petroleum prices rose sharply from a previously relatively stable level. Table 3 shows that Ghana typically enjoys an excellent hedging basis with the world cocoa price, and the same must also be true of gold where we do not have the unit value data required to undertake that calculation. By contrast, prices only explain around 22% of changes in cocoa revenues and 45% of changes in gold revenues.

Ghanaian gold export revenues exhibit a weak negative correlation with the world cocoa price ( $r = -0.10$ ) while gold exports show a positive correlation with the gold price ( $r = 0.46$ ). Oil import expenditures were weakly negatively correlated with the world oil

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<sup>14</sup> We infer the value of Ghanaian gold exports by valuing production (reported in World Bureau of Metal Statistics, World Metal Statistics, various issues) at the average London gold price. The implicit assumption is that domestic gold consumption is negligible.

price ( $r = -0.13$ ). The low cocoa correlation reflects the hedging practices of Cocobod, the monopoly-monopsony state cocoa marketing board, which sells around three quarters of the crop forward.<sup>15</sup> These correlations imply that any hedge should primarily relate to gold and not to cocoa or oil. The regression-based hedge ratios are 42% for gold (close to its export share), 8% for cocoa (much less than its export share) and zero for petroleum, where the regression estimates would have implied a negative hedge ratio.

Figure G4 compares Ghana's simulated concessional debt repayments under the export and debt service-based swap schemes A (using regression weights) compared with historic repayments. Payments are lower in eight of the years and higher in just three years. The payments under the debt-service based scheme move in the same way as those under those in which trend exports are the instrument, but the impacts are much smaller. The relatively low ratio of concessional debt service to exports is sufficient to cause the welfare impact of the debt service-based scheme to be negligible – see appendix Table A1.

Finally, Figure G5 graphs the resulting quantity of free foreign exchange in the two simulated environments. Export revenues grew faster than oil import and debt service expenditures through the first half of the nineteen nineties releasing foreign exchange for other uses. This trend came to a stop in 1996, with export revenues dropping to a lower level for the remainder of the decade. This fall was partly due to a fall in cocoa prices and to lower gold prices, but also partly reflected high oil consumption in what had become a richer and more successful economy, and to the fact that the rate of growth of gold production had declined as mine expansion plans were completed. These changes appear to be as much structural as due to cyclical price movements, and schemes driven by cyclical price movements therefore only offer modest improvements.

The overall conclusion of the analysis is that any difficulties Ghana may have experienced in meeting debt service payments on concessional debt in the nineteen nineties, and we are not aware that were such difficulties, were not primarily due to movements in its export or import prices. In this light, it is unsurprising that the schemes we have considered fail to generate much value.

#### Country study: Benin

Benin gives the poorest results of all. Benin's major export is cotton, which accounted for an average 35% of its total export revenues over our simulation period of 1991-2000. The cost of oil imports averaged 16% of total export revenues over the same period. Appendix Figure N1 graphs these receipts and expenditures. Note that we estimated oil import figures by linear interpolation over the period 1991-93.<sup>16</sup> The data show both cotton revenues and oil import costs to be highly variable.

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<sup>15</sup> Ashanti Gold Fields Corporation, the major private sector gold mining company, also sells its gold forward but this will not be evident in the way we have imputed gold revenues – see previous footnote.

<sup>16</sup> Data on the quantities of oil imported are available for these years, but we judged them to be incredible both in terms of their absolute levels and relative to the reported value of total imports.

Appendix Figure N2 shows Benin's concessional debt service over the period 1984-2001. Concessional debt repayments have grown over time to reach a pre-HIPC maximum of 11% of export revenue in 2000. The African Development Bank is as important as IDA as a recipient of concessional debt repayment.

Appendix Figure N3 graphs the cotton cocoa price (left hand axis) and petroleum price (right hand axis) over the simulation period. The cotton price is an index based on a survey of supposedly representative transactions, and it is not clear whether this will correspond closely to then price obtained by Benin's ginneries. The cotton price achieved its highest level in 1995 since which time it has been depressed. As already noted, oil prices rose sharply in 2000 and 2001. The Benin cotton price is poorly correlated with our world price – see Table 3 – and movements in the world price explain just 20% of Benin's cotton revenues – see Table 4.

Benin's cotton revenues show only a modest positive correlation with the international cotton price ( $r = 0.33$ ) while oil import expenditures were weakly negatively correlated with the world oil price ( $r = -0.22$ ). These correlations imply that any hedge should primarily relate to cotton rather than oil. However, the regression-based hedge ratios are 35% for gold, 16% for oil, close to their shares of total exports.

Figure N4 compares Benin's simulated concessional debt repayments under the export and debt service-based swap schemes A (using regression weights) compared with historic repayments. Payments are lower in five of the years and higher in just two years, with the remaining four years showing little change. The payments under the debt-service based scheme move in the same way as those under those in which trend exports are the instrument, but the impacts are much smaller.

Finally, Figure N5 graphs the resulting quantity of free foreign exchange in the two simulated environments. Movements in this quantity are dominated by the low initial figure in 1991 and the high spike in 1996. Neither of these large movements appears related to either the cotton or the oil price. In the absence of alternative explanations, we believe they may primarily result from poor quality national accounting data rather than to any underlying real movement in the economy.<sup>17</sup> Furthermore, Benin's cotton unit values are poorly correlated with world prices. For these reasons, we are not persuaded that the negative outcomes in the Benin analysis reflect more than inadequate data quality. We therefore simply note that data of reasonable quality are a prerequisite for any form of informed policy analysis.

### Summary

Each of the three countries we have considered tells a different story. The common element is that movements in international commodity prices are not, in general, the

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<sup>17</sup> We believe this particularly for the 1996 spike which reflects an anomalously high reported value for total exports. This observation remains true even after a recent large downward adjustment of a previously published figure.

major factor responsible for movements in reported export revenues and import expenditures, even in countries which are highly dependent on these commodities. This is partly because of basis risk and partly because of unrelated quantity variations.

- In Burundi, where coffee prices do account for a significant proportion of the variation of coffee revenues, there were two major falls in revenue over the nineteen nineties which were not due to price movements.
- In Ghana, movements in commodity export and oil import revenues relate more to structural factors as to price movements.
- In Benin, data quality issues are sufficiently acute as to lead to the worry that the results may be dominated by accounting errors, preventing any clear substantive conclusion beyond the concern that the local and world cotton prices appear too weakly related for a hedge scheme to be viable.

None of this implies that price effects are unimportant. Rather, the arguments underline that these are not the only important factor, or that the data are insufficient to allow secure judgments. But these factors do explain why schemes, such as those we consider, which rely only on prices, may be less effective than one might have hoped.

## **7. Conclusions**

We have asked the question of whether it is possible to increase the flexibility of heavily indebted primary producing countries in meeting their concessional debt service obligations. The answer to this question is that this can in principle be done by augmenting existing concessional debt agreements by a set of floating-for-fixed commodity swaps or swaptions. Importantly, these should be “variable strike” instruments in which the “fixed” price is an estimate of the underlying trend price, probably implemented through a moving average of past prices. Modifications of this sort certainly appear feasible.

Feasibility does not imply effectiveness. Countries may already have methods for coping with commodity price variability, even at the macroeconomic level, and these may be undermined by hedging arrangements. Of even greater importance, different countries are subject to a large number of shocks (for example, demand shocks, exchange rate changes and political disturbances), and the effects of these shocks may dominate those of commodity price shocks. The issue of whether a particular country would have benefited from a scheme of this sort, for example over the past decade, is therefore primarily empirical. We have investigated this for ten commodity dependent severely or moderately indebted African countries.

The scale of the benefits varies from country to country. Using a fairly conservative money metric, the proposed instrument turns out to be worth the same as a cut in debt service payments of between zero and 5%. The median value across our sample is between ½% and 2¼% depending on scheme design. Use of a less conservative risk aversion parameter could raise these estimates by a factor of two or, conceivably, three. However, those scheme which are seen as generating the highest (1%-2¼%) benefits adopt hedge positions in relation to trend exports rather than contractual debt service commitments, and the additional benefits generated by these schemes turns out to be at

the expense of a disproportionately large disruption of debt service payments to the concessionary lenders, and in particular to IDA. One is therefore confronted by a choice between schemes which impose relatively low costs on the lenders but generate correspondingly low benefits to the borrowers, and alternatives which generate higher, albeit still modest, benefits, but at the expense of very considerable and potentially costly disruption of flows to lenders.

We have looked in some detail at the reasons for the relatively weak performance of these price-based schemes. The main reason appears to be that movements in international prices are only one of several factors that generate movements in countries' reported commodity earnings. In general, world prices account for less than half of total revenue movements. Quantity variations and variations in local prices relative to those in world markets also play an important role. Schemes, such as those we consider, which are based on world prices cannot offer protection against these other factors.

As noted in the introduction, this paper has set out to analyze the impacts of schemes to link repayment of concessional debt to ability to pay and not to make a recommendation on adoption of such a scheme. We therefore forbear from making such a recommendation. However, we note that, to the extent that our results are judged negative, this does suggest that renewed attention should be given to schemes which focus on countries export earnings rather than at export prices despite the well known problems (timeliness, moral hazard etc) that arise in connection with these schemes.

## Appendix: Commodity Hedge Parameters and Detailed Simulation Results

### General Comments and Notation

Write the country's debt service level as  $S$ , its level of export earnings as  $X$  and its level of oil import expenditures as  $O$ . There is no general reason to suppose current account balance and in many developing countries imports are financed as much aid and other capital inflows. Write  $\bar{X} = E[X]$  and  $\bar{O} = E[O]$ . Define  $\psi = \frac{\bar{S}}{\bar{X}}$ , the ratio of average concessional debt service to total export revenues. The export commodity on average constitutes  $\alpha\%$  of the country's total export earnings, and the import commodity  $\gamma\%$  of its export expenditures. Write revenues from the identified export commodity as  $X_1$  and all those from other exports as  $X_2$ . Total export receipts are  $X = X_1 + X_2$ . This allows us to

approximate  $x = \alpha x_1 + (1 - \alpha)x_2$ , where  $x_j = \frac{X_j - E(X_j)}{\bar{X}}$ , ( $j=1,2$ ). Similarly, write

$x_0 = \frac{O - \bar{O}}{\bar{X}}$ . Let the commodity export price be  $P$  and the oil import price be  $Q$ , and

write  $p = \frac{P - \bar{P}}{\bar{P}}$  and  $q = \frac{Q - \bar{Q}}{\bar{Q}}$  where  $\bar{P} = E(P)$  and  $\bar{Q} = E(Q)$ . Because prices trend,

the expectations  $\bar{P}$  and  $\bar{Q}$  are to be interpreted as conditional – in practice, we will use moving averages. Write  $Var(x_i) = \sigma_i^2$  ( $i = 0,1,2$ ),  $Cov(p, x_i) = \rho_{pi} \sigma_p \sigma_i$  ( $i = 0,1,2$ ) and

$Cov(q, x_i) = \rho_{qi} \sigma_q \sigma_i$  ( $i = 0,1,2$ ). Write  $Var\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} \sigma_p^2 & \rho_{pq} \sigma_p \sigma_q \\ \rho_{pq} \sigma_p \sigma_q & \sigma_q^2 \end{pmatrix}$ .

In the absence of the scheme, debt service would be  $\bar{S}$ . We consider a scheme in which debt service are augmented by  $\lambda \bar{X}$  fixed for floating swaps defined in terms of the export price  $P$  and  $\mu \bar{X}$  reverse fixed for floating swaps defined in terms of the oil import price  $Q$ . The scheme modifies debt service to

$$S = \bar{S} + (\lambda p - \mu q) \bar{X} \quad (A1)$$

The objective is to determine the optimal swap factors  $\lambda$  and  $\mu$ . For simplicity, we ignore the cap and floor or the swap payments discussed in the text.

To obtain an optimal hedge, in terms of a hedge which minimizes an appropriate measure of variability, we consider first minimization of the variance  $Var(X - O - S)$ .

Minimization of this variance is equivalent to minimization of

$$Var(S) - 2Cov(S, X) + Cov(S, O) \quad (A2)$$

We may evaluate

$$Var(S) = (\lambda^2 \sigma_p^2 - 2\lambda\mu\rho_{pq} \sigma_p \sigma_q + \mu^2 \sigma_q^2) \bar{X}^2 \quad (A3)$$

Turning to the covariance terms in equation (A2),

$$Cov(S, X) = \left\{ \lambda \sigma_p \left[ \alpha \rho_{p1} \sigma_1 + (1 - \alpha) \rho_{p2} \sigma_2 \right] - \mu \sigma_q \left[ \alpha \rho_{q1} \sigma_1 + (1 - \alpha) \rho_{q2} \sigma_2 \right] \right\} \bar{X}^2 \quad (A4)$$

$$Cov(S, O) = \left( \lambda \rho_{p0} \sigma_p \sigma_0 - \mu \rho_{q0} \sigma_q \sigma_0 \right) \bar{X}^2 \quad (A5)$$

Substituting equations (A3-A5) into equation (A2) gives the criterion function as

$$\begin{aligned} & \left( \lambda^2 \sigma_p^2 - 2\lambda \mu \rho_{pq} \sigma_p \sigma_q + \mu^2 \sigma_q^2 \right) - 2 \left( \lambda \rho_{p0} \sigma_p \sigma_0 - \mu \rho_{q0} \sigma_q \sigma_0 \right) \\ & - 2 \left\{ \lambda \sigma_p \left[ \alpha \rho_{p1} \sigma_1 + (1 - \alpha) \rho_{p2} \sigma_2 \right] - \mu \sigma_q \left[ \alpha \rho_{q1} \sigma_1 + (1 - \alpha) \rho_{q2} \sigma_2 \right] \right\} \end{aligned} \quad (A6)$$

Minimizing expression (A6) with respect to  $\lambda$

$$\lambda \sigma_p^2 - \mu \rho_{pq} \sigma_p \sigma_q = \sigma_p \left\{ \left[ \alpha \rho_{p1} \sigma_1 + (1 - \alpha) \rho_{p2} \sigma_2 \right] - \rho_{p0} \sigma_0 \right\} \quad (A7)$$

Similarly, minimizing with respect to  $\mu$ ,

$$\mu \sigma_q^2 - \lambda \rho_{pq} \sigma_p \sigma_q = -\sigma_q \left\{ \left[ \alpha \rho_{q1} \sigma_1 + (1 - \alpha) \rho_{q2} \sigma_2 \right] - \rho_{q0} \sigma_0 \right\} \quad (A8)$$

We may summarize equations (A7) and (A8) as

$$\begin{pmatrix} \sigma_p^2 & -\rho_{pq} \sigma_p \sigma_q \\ -\rho_{pq} \sigma_p \sigma_q & \sigma_q^2 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} \sigma_p \kappa_p \\ \sigma_q \kappa_q \end{pmatrix} \quad (A9)$$

where  $\kappa_p = \left[ \alpha \beta_{1p} + (1 - \alpha) \beta_{2p} \right] - \beta_{0p}$  where  $\beta_{jp} = \frac{\rho_{pj} \sigma_j}{\sigma_p}$ , the simple regression

coefficient of  $x_j$  on  $p$  ( $j = 0, 1, 2$ ), and  $\kappa_q = -\left[ \alpha \beta_{1q} + (1 - \alpha) \beta_{2q} \right] + \beta_{0q}$  where

$\beta_{jq} = \frac{\rho_{qj} \sigma_j}{\sigma_q}$ , the simple regression coefficient of  $x_j$  on  $q$  ( $j = 0, 1, 2$ ). Inverting the matrix

on the left hand side of equation (A9), we obtain

$$\lambda = \frac{\kappa_p - \rho_{pq} \kappa_q}{1 - \rho_{pq}^2} \quad (A10)$$

and

$$\mu = \frac{\kappa_q - \rho_{pq} \kappa_p}{1 - \rho_{pq}^2} \quad (A11)$$

This is equivalent to regression of  $\frac{X - O}{\bar{X}}$  on  $p$  and  $q$ .

In the most simple case we could expect unit hedge ratios for the export commodity and oil respectively ( $\beta_{1p} = \beta_{0q} = 1$ ) with no benefits from taking the portfolio view – i.e.

$\beta_{0p} = \beta_{2p} = \beta_{1q} = \beta_{2q} = 0$  and  $\rho_{pq} = 0$ . In that case, equations (A10) and (A11) simply give the commodity shares, ie  $\lambda = \alpha$  and  $\mu = \gamma$ .

We now turn to welfare analysis. Let social welfare  $W$  have three components:

- the expected utility from consumption  $C$  of subsistence goods  $Z$ ,
- the expected utility obtained from imported consumer goods  $M$ , and
- the expected utility of consumption of oil imports.

We suppose that the social welfare function is additively separable between the first of these components and the remaining two.

$$W = EU(M) + EV(Z, O) \quad (A12)$$

Ignoring, for simplicity, remittances, aid and capital inflows, the balance of payments constraint is  $X = M + O + S$  social welfare becomes

$$W(\lambda, \mu) = EU(X - S(\lambda, \mu) - O) + EV(Z, O) \quad (A13)$$

Maximization of social welfare with respect to the two parameters  $\lambda$  and  $\mu$  is equivalent to maximization of  $EU(X - S(\lambda, \mu) - O)$ . The assumption of additive separability implies that we do not need to consider the second right hand side term in equation (A13).

Write the expected utility level obtained in the absence of intervention (i.e.  $\lambda = \mu = 0$ ) as  $u_0$ . For any pair of values of  $\lambda$  and  $\mu$ , we may define the reduction  $\delta(\lambda, \mu)$  in debt service that the country would regard as equivalent, in social welfare terms, to the reduction in variability resulting from the choice of non-zero values for  $\lambda$  and  $\mu$ . This reduction is defined implicitly by the equation

$$EU(X - S(\lambda, \mu) - O) = EU(X - (1 - \delta)S_0 - O) \quad (A14)$$

where  $S_0 = S(0, 0)$ , the non-intervention level of debt service. Expanding (A14) around  $\lambda = \mu = 0$ , we obtain

$$\frac{1}{2}u_0'' Var[X - S(\lambda, \mu) - O] = \frac{1}{2}u_0'' Var[X - S_0 - O] + u_0' \delta S_0 \quad (A15)$$

Write the reduction (measured as positive) in the proportionate variance as

$$\Delta = \frac{Var[X - S_0 - O] - Var[X - S(\lambda, \mu) - O]}{\bar{X}^2}$$

where  $\bar{X} = EX$ , the expected level of exports, which we shall interpret as the moving average trend. Then

$$\delta = \frac{R\Delta}{2\psi} \quad (A16)$$

where  $\psi = \frac{S_0}{\bar{X}}$ , the ratio of debt service to exports, and  $R = -\frac{\bar{X}u_0''}{u_0'}$ , the coefficient of partial risk aversion.

The coefficient of partial risk aversion is related to the more standard coefficient of relative risk aversion  $\rho$  as  $R = \frac{\rho\bar{X}}{\bar{Y}}$  where  $\bar{Y} = EY$ , the expected level of income. The equivalent expression for  $\delta$  in terms of the relative risk aversion coefficient is

$$\delta = \frac{\rho\Delta}{2\xi} \quad (A17)$$

where  $\xi = \frac{S_0}{\bar{Y}}$ , the ratio of debt service to income. Equations (A16) and (A17) are equivalent. the substantive issue is whether it is more plausible to regard partial or

relative risk aversion as constant across countries with different export to GDP ratios. We prefer to work with constant partial risk aversion on the argument that the export to GDP ratio should not be a major determinant of the debt reduction equivalent of a reduction in the variability of the debt service to export ratio. But recognizing that partial risk aversion must be lower than relative risk aversion, we take a uniform low value of unity for this parameter.

**Table A1**  
**Simulation Summary Statistics (1991-2000)**

Objective: Minimization of $Var(X - O - S)$			Instrument: $S$					
	Actual	Scheme	Share-based Weights			Regression-based weights		
	standard deviation		standard deviation	welfare change %	welfare change \$m	standard deviation	welfare change %	welfare change \$m
<b>Benin</b>	17.5%	A	17.6%	-0.2%	-\$0.1m	17.6%	-0.2%	-\$0.1m
		B	17.6%	-0.1%	-\$0.0m	17.6%	-0.1%	-\$0.0m
		C	17.5%	-0.0%	-\$0.0m	17.5%	-0.0%	-\$0.0m
<b>Burkina Faso</b>	35.9%	A	35.1%	1.6%	\$0.5m	34.5%	2.6%	\$0.8m
		B	35.3%	1.2%	\$0.3m	35.0%	1.8%	\$0.5m
		C	35.4%	1.0%	\$0.3m	34.6%	2.5%	\$0.7m
<b>Burundi</b>	28.2%	A	23.1%	5.1%	\$1.0m	24.7%	3.6%	\$0.7m
		B	24.1%	4.2%	\$0.8m	25.5%	2.8%	\$0.5m
		C	23.4%	5.0%	\$1.0m	24.8%	3.5%	\$0.7m
<b>Cameroon (1991-99)</b>	9.5%	A	9.3%	0.4%	\$0.3m	9.2%	0.6%	\$0.5m
		B	9.5%	0.0%	\$0.0m	9.4%	0.1%	\$0.1m
		C	9.5%	0.0%	\$0.0m	9.4%	0.2%	\$0.1m
<b>Ghana</b>	20.0%	A	19.9%	0.3%	\$0.3m	20.0%	0.1%	\$0.1m
		B	20.0%	0.1%	\$0.1m	20.0%	-0.0%	-\$0.0m
		C	19.9%	0.3%	\$0.3m	20.0%	0.0%	\$0.0m
<b>Kenya</b>	5.3%	A	5.0%	0.2%	\$0.2m	5.0%	0.2%	\$0.2m
		B	5.0%	0.2%	\$0.2m	5.1%	0.2%	\$0.2m
		C	5.3%	-0.0%	-\$0.0m	5.3%	0.0%	\$0.0m
<b>Madagascar</b>	26.6%	A	26.5%	0.2%	\$0.1m	26.0%	1.0%	\$0.5m
		B	26.7%	-0.1%	-\$0.1m	26.2%	0.6%	\$0.3m
		C	27.0%	-0.5%	-\$0.3m	26.1%	0.9%	\$0.4m
<b>Malawi</b>	12.3%	A	11.6%	0.9%	\$0.3m	11.8%	0.7%	\$0.2m
		B	12.1%	0.2%	\$0.1m	12.2%	0.1%	\$0.0m
		C	12.1%	0.2%	\$0.1m	12.3%	0.0%	\$0.0m
<b>Rwanda (excludes 1994-95)</b>	19.1%	A	15.8%	3.5%	\$0.5m	16.0%	3.2%	\$0.5m
		B	17.4%	1.8%	\$0.3m	18.4%	0.8%	\$0.1m
		C	17.0%	2.3%	\$0.3m	16.3%	2.9%	\$0.4m
<b>Tanzania</b>	12.1%	A	11.6%	0.4%	\$0.3m	11.6%	0.4%	\$0.3m
		B	11.7%	0.3%	\$0.2m	11.9%	0.1%	\$0.1m
		C	11.8%	0.2%	\$0.2m	12.1%	0.0%	\$0.0m
<b>Average</b>	19.4%	A	18.3%	1.2%	\$0.3m	18.6%	1.0%	\$0.3m
		B	18.7%	0.8%	\$0.2m	19.0%	0.5%	\$0.1m
		C	18.6%	0.9%	\$0.2m	18.9%	0.8%	\$0.2m
<b>Median</b>	19.5%	A	17.9%	0.4%	\$0.3m	18.0%	0.7%	\$0.3m
		B	18.7%	0.2%	\$0.2m	19.2%	0.2%	\$0.1m
		C	18.4%	0.2%	\$0.2m	18.2%	0.2%	\$0.1m

The standard deviation measures the standard deviation of the approximated ratio of debt service to exports, less a proportion of oil imports – see text.

The welfare change is calculated as the percentage and dollar values of the reduction in concessional debt service which would yield the same welfare improvement as the proposed modifications to concessional debt contracts. All welfare changes assume a unit coefficient of partial risk aversion. They increase proportionately for higher values of this coefficient.

**Table A2**  
**Simulation Summary Statistics (1991-2000)**

Objective: Minimization of $Var\left(\frac{S + \gamma O}{X}\right)$			Instrument: $S$					
	Actual	Scheme	Share-based Weights			Regression-based weights		
	standard deviation		standard deviation	welfare change %	welfare change \$m	standard deviation	welfare change %	welfare change \$m
<b>Benin</b>	4.5%	A	4.6%	-0.3%	-\$0.1m	4.5%	-0.1%	-\$0.0m
		B	4.5%	-0.0%	-\$0.0m	4.5%	-0.0%	-\$0.0m
		C	4.5%	0.0%	\$0.0m	4.5%	0.0%	\$0.0m
<b>Burkina Faso</b>	10.1%	A	9.5%	1.6%	\$0.5m	9.7%	0.6%	\$0.2m
		B	9.6%	1.2%	\$0.3m	10.0%	0.3%	\$0.1m
		C	10.0%	1.0%	\$0.3m	10.1%	0.0%	\$0.0m
<b>Burundi</b>	17.3%	A	13.9%	5.1%	\$1.0m	16.1%	1.7%	\$0.3m
		B	14.4%	4.2%	\$0.8m	16.4%	1.3%	\$0.3m
		C	14.3%	5.0%	\$1.0m	17.3%	0.0%	\$0.0m
<b>Cameroon (1991-99)</b>	1.4%	A	1.2%	0.4%	\$0.3m	1.3%	1.5%	\$1.1m
		B	1.3%	0.0%	\$0.0m	1.2%	0.2%	\$0.2m
		C	1.4%	0.0%	\$0.0m	1.2%	1.3%	\$1.0m
<b>Ghana</b>	5.5%	A	5.4%	0.3%	\$0.3m	5.5%	0.0%	\$0.0m
		B	5.5%	0.1%	\$0.1m	5.5%	0.0%	\$0.0m
		C	5.4%	0.3%	\$0.3m	5.5%	0.0%	\$0.0m
<b>Kenya</b>	2.1%	A	1.8%	0.2%	\$0.2m	2.1%	0.0%	\$0.0m
		B	1.9%	0.2%	\$0.2m	2.1%	0.0%	\$0.0m
		C	2.1%	-0.0%	-\$0.0m	2.1%	0.0%	\$0.0m
<b>Madagascar</b>	12.4%	A	12.4%	0.2%	\$0.1m	12.3%	0.4%	\$0.2m
		B	12.6%	-0.1%	-\$0.1m	12.4%	0.2%	\$0.1m
		C	12.8%	-0.5%	-\$0.3m	12.4%	0.0%	\$0.0m
<b>Malawi</b>	1.5%	A	1.3%	0.9%	\$0.3m	1.4%	0.0%	\$0.0m
		B	1.4%	0.2%	\$0.1m	1.5%	0.0%	\$0.0m
		C	1.5%	0.2%	\$0.1m	1.5%	0.0%	\$0.0m
<b>Rwanda (excludes 1994-95)</b>	5.9%	A	3.4%	3.5%	\$0.5m	5.6%	0.5%	\$0.1m
		B	4.4%	1.8%	\$0.3m	5.8%	0.1%	\$0.0m
		C	5.0%	2.3%	\$0.3m	5.9%	0.0%	\$0.0m
<b>Tanzania</b>	5.0%	A	4.4%	0.4%	\$0.3m	4.8%	0.1%	\$0.1m
		B	4.5%	0.3%	\$0.2m	4.9%	0.1%	\$0.0m
		C	4.4%	0.2%	\$0.2m	5.0%	0.0%	\$0.0m
<b>Average</b>	6.6%	A	5.8%	1.2%	\$0.3m	6.3%	0.8%	\$0.2m
		B	6.0%	0.8%	\$0.2m	6.4%	0.8%	\$0.1m
		C	6.1%	0.9%	\$0.2m	6.6%	0.1%	\$0.1m
<b>Median</b>	5.3%	A	4.5%	0.4%	\$0.3m	5.2%	0.2%	\$0.1m
		B	4.5%	0.2%	\$0.2m	4.5%	0.2%	\$0.0m
		C	4.8%	0.2%	\$0.2m	5.3%	0.0%	\$0.0m

The standard deviation measures the standard deviation of the approximated ratio of debt service to exports, less a proportion of oil imports – see text.

The welfare change is calculated as the percentage and dollar values of the reduction in concessional debt service which would yield the same welfare improvement as the proposed modifications to concessional debt contracts. All welfare changes assume a unit coefficient of partial risk aversion. They increase proportionately for higher values of this coefficient.

**Table A3**  
**Simulation Summary Statistics (1991-2000)**

Objective: Minimization of $Var(X - O - S)$			Instrument: $\tilde{X}$					
	Actual	Scheme	Share-based Weights			Regression-based weights		
	standard deviation		standard deviation	welfare change %	welfare change \$m	standard deviation	welfare change %	welfare change \$m
<b>Benin</b>	17.5%	A	19.0%	-2.9%	-\$0.8m	18.8%	-2.6%	-\$0.7m
		B	18.4%	-1.8%	-\$0.5m	18.4%	-1.7%	-\$0.5m
		C	17.5%	0.0%	\$0.0m	17.5%	0.0%	\$0.0m
<b>Burkina Faso</b>	35.9%	A	33.6%	4.2%	\$1.2m	34.0%	3.6%	\$1.1m
		B	33.6%	4.4%	\$1.3m	32.3%	6.6%	\$1.9m
		C	35.5%	0.8%	\$0.2m	34.1%	3.4%	\$1.0m
<b>Burundi</b>	28.2%	A	25.7%	2.7%	\$0.5m	23.6%	4.7%	\$0.9m
		B	22.5%	5.6%	\$1.1m	22.3%	5.8%	\$1.1m
		C	26.2%	2.1%	\$0.4m	22.4%	5.8%	\$1.1m
<b>Cameroon (1991-99)</b>	9.5%	A	8.8%	1.5%	\$1.1m	7.7%	3.3%	\$2.5m
		B	9.7%	-0.5%	-\$0.3m	9.0%	1.0%	\$0.8m
		C	9.5%	0.0%	\$0.0m	9.4%	0.3%	\$0.2m
<b>Ghana</b>	20.0%	A	19.9%	0.2%	\$0.2m	19.7%	0.8%	\$0.8m
		B	20.0%	0.2%	\$0.2m	20.1%	-0.1%	-\$0.2m
		C	19.7%	0.8%	\$0.8m	20.0%	0.0%	\$0.0m
<b>Kenya</b>	53%	A	4.8%	0.3%	\$0.4m	4.6%	0.4%	\$0.5m
		B	4.0%	0.8%	\$0.8m	3.8%	0.9%	\$0.9m
		C	5.7%	-0.3%	-\$0.3m	5.3%	0.0%	\$0.0m
<b>Madagascar</b>	26.6%	A	27.6%	-1.5%	-\$0.7m	24.6%	3.0%	\$1.5m
		B	26.8%	-0.3%	-\$0.2m	25.1%	2.4%	\$1.2m
		C	26.8%	-0.3%	-\$0.2m	25.1%	2.4%	\$1.2m
<b>Malawi</b>	12.3%	A	8.0%	5.4%	\$1.6m	8.2%	5.2%	\$1.6m
		B	10.5%	2.5%	\$0.7m	11.4%	1.2%	\$0.4m
		C	11.7%	0.8%	\$0.2m	12.3%	0.0%	\$0.0m
<b>Rwanda (excludes 1994-95)</b>	19.1%	A	12.9%	5.9%	\$0.9m	13.1%	5.7%	\$0.9m
		B	13.9%	5.1%	\$0.8m	15.4%	3.8%	\$0.6m
		C	13.3%	5.6%	\$0.9m	13.3%	5.6%	\$0.8m
<b>Tanzania</b>	12.1%	A	11.6%	0.4%	\$0.3m	10.0%	1.6%	\$1.3m
		B	11.1%	0.7%	\$0.6m	11.1%	0.8%	\$0.7m
		C	12.1%	-0.0%	-\$0.0m	12.1%	0.0%	\$0.0m
<b>Average</b>	19.4%	A	17.9%	1.6%	\$0.5m	17.1%	2.8%	\$1.1m
		B	17.8%	1.6%	\$0.4m	17.8%	2.0%	\$0.7m
		C	18.5%	1.0%	\$0.2m	18.3%	1.4%	\$0.4m
<b>Median</b>	19.5%	A	16.4%	1.0%	\$0.5m	16.4%	2.2%	\$1.0m
		B	17.0%	0.8%	\$0.7m	17.8%	1.1%	\$0.7m
		C	16.5%	0.4%	\$0.1m	16.7%	0.2%	\$0.1m

The standard deviation measures the standard deviation (about its moving average trend) of the difference between exports and the sum of debt service and oil imports – see text.  
The welfare change is calculated as the percentage and dollar values of the reduction in concessional debt service which would yield the same welfare improvement as the proposed modifications to concessional debt contracts. All welfare changes assume a unit coefficient of partial risk aversion. They increase proportionately for higher values of this coefficient.

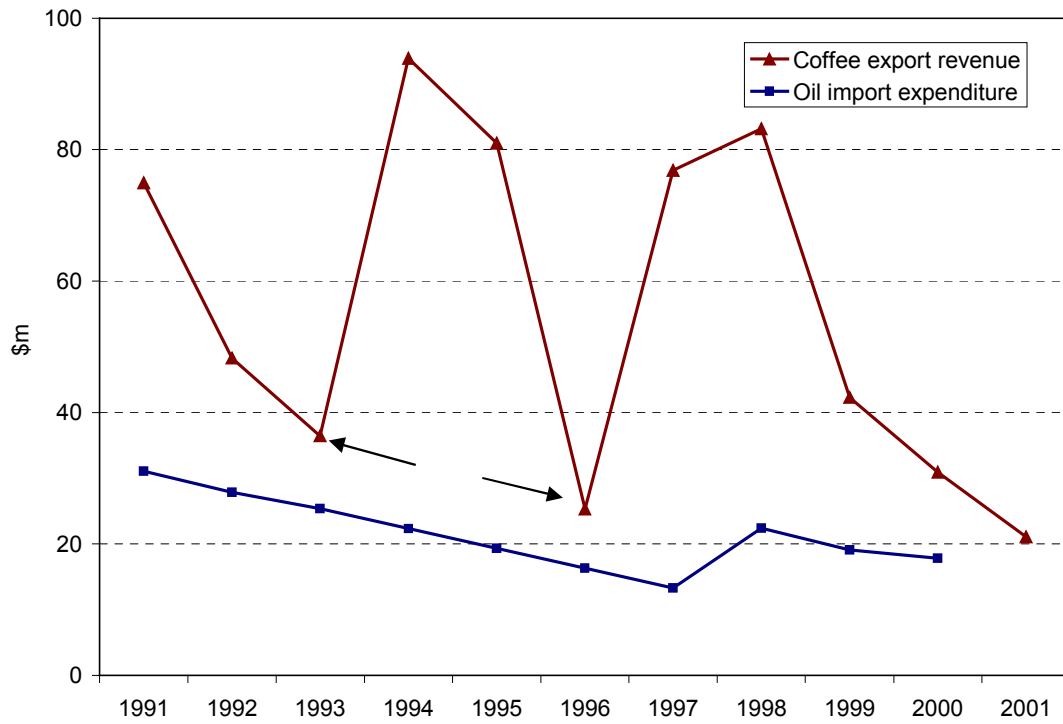
**Table A4**  
**Simulation Summary Statistics (1991-2000)**

Objective: Minimization of $Var\left(\frac{S + \gamma O}{X}\right)$			Instrument: $\tilde{X}$					
	Actual	Scheme	Share-based Weights			Regression-based weights		
	standard deviation		standard deviation	welfare change %	welfare change \$m	standard deviation	welfare change %	welfare change \$m
<b>Benin</b>	4.5%	A	7.0%	-3.4%	-\$0.9m	4.5%	-0.0%	-\$0.0m
		B	5.9%	-2.2%	-\$0.6m	4.5%	-0.0%	-\$0.0m
		C	4.5%	0.0%	\$0.0m	4.5%	0.0%	\$0.0m
<b>Burkina Faso</b>	10.1%	A	10.2%	4.2%	\$1.2m	8.2%	2.8%	\$0.8m
		B	8.7%	4.4%	\$1.3m	9.3%	1.4%	\$0.4m
		C	10.5%	0.8%	\$0.2m	10.1%	0.0%	\$0.0m
<b>Burundi</b>	17.3%	A	19.6%	2.7%	\$0.5m	14.3%	4.9%	\$0.9m
		B	15.5%	5.6%	\$1.1m	14.2%	4.4%	\$0.8m
		C	19.7%	2.1%	\$0.4m	15.9%	2.9%	\$0.6m
<b>Cameroon (1991-99)</b>	1.4%	A	3.3%	1.5%	\$1.1m	1.4%	1.3%	\$1.0m
		B	2.0%	-0.5%	-\$0.3m	1.2%	0.2%	\$0.2m
		C	1.4%	0.0%	\$0.0m	1.4%	0.0%	\$0.0m
<b>Ghana</b>	5.5%	A	7.0%	0.2%	\$0.2m	5.3%	0.1%	\$0.1m
		B	6.1%	0.2%	\$0.2m	5.5%	-0.0%	-\$0.0m
		C	5.8%	0.8%	\$0.8m	5.5%	0.0%	\$0.0m
<b>Kenya</b>	2.1%	A	3.7%	0.3%	\$0.4m	1.6%	0.3%	\$0.3m
		B	3.2%	0.8%	\$0.8m	1.8%	0.2%	\$0.2m
		C	3.0%	-0.3%	-\$0.3m	2.1%	0.0%	\$0.0m
<b>Madagascar</b>	12.4%	A	15.0%	-1.5%	-\$0.7m	11.0%	3.3%	\$1.6m
		B	13.9%	-0.3%	-\$0.2m	11.4%	2.8%	\$1.4m
		C	13.9%	-0.3%	-\$0.2m	11.4%	2.8%	\$1.4m
<b>Malawi</b>	1.5%	A	6.1%	5.4%	\$1.6m	1.1%	0.2%	\$0.1m
		B	2.5%	2.5%	\$0.7m	1.2%	0.2%	\$0.1m
		C	2.9%	0.8%	\$0.2m	1.5%	0.0%	\$0.0m
<b>Rwanda (excludes 1994-95)</b>	5.9%	A	9.7%	5.9%	\$0.9m	5.0%	1.5%	\$0.2m
		B	5.6%	5.1%	\$0.8m	5.6%	0.4%	\$0.1m
		C	5.6%	5.6%	\$0.9m	5.9%	0.0%	\$0.0m
<b>Tanzania</b>	5.0%	A	7.5%	0.4%	\$0.3m	4.1%	0.7%	\$0.6m
		B	5.7%	0.7%	\$0.6m	4.5%	0.3%	\$0.3m
		C	6.3%	-0.0%	-\$0.0m	5.0%	0.0%	\$0.0m
<b>Average</b>	6.6%	A	8.9%	1.6%	\$0.5m	5.7%	1.5%	\$0.6m
		B	6.9%	1.6%	\$0.4m	5.9%	1.6%	\$0.4m
		C	7.4%	1.0%	\$0.2m	6.3%	0.6%	\$0.2m
<b>Median</b>	5.3%	A	7.3%	1.0%	\$0.5m	4.8%	1.0%	\$0.4m
		B	5.8%	0.8%	\$0.7m	5.0%	0.3%	\$0.2m
		C	5.7%	0.4%	\$0.1m	5.3%	0.0%	\$0.0m

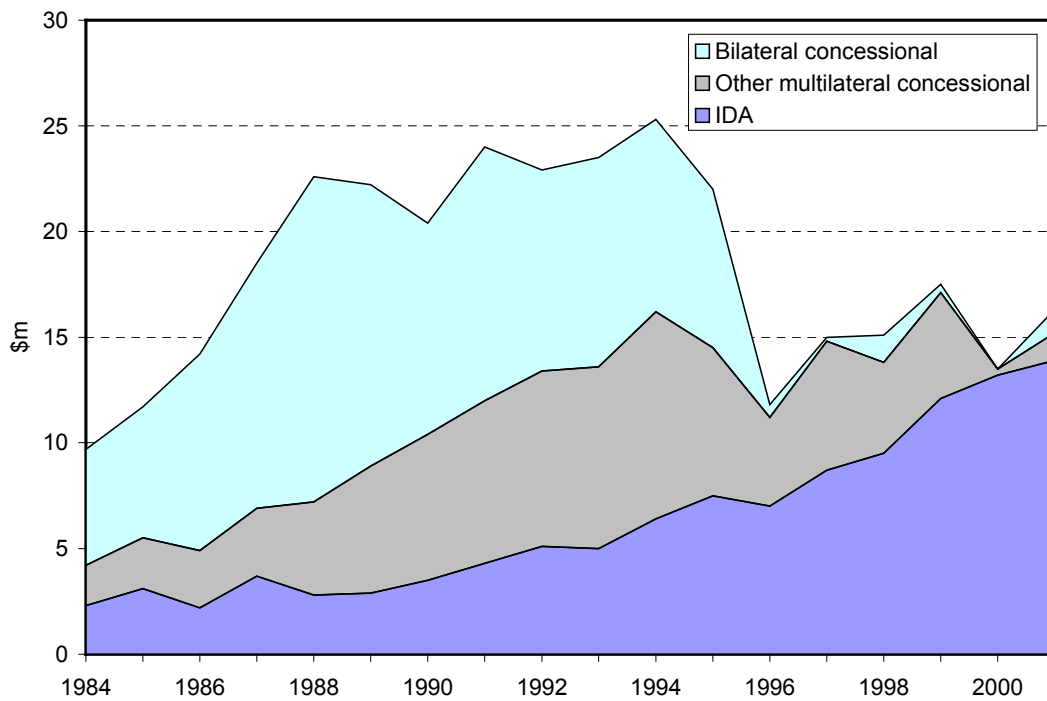
The standard deviation measures the standard deviation (measured about its moving average trend) of the ratio of debt service to exports, less a proportion of oil imports – see text.

The welfare change is calculated as the percentage and dollar values of the reduction in concessional debt service which would yield the same welfare improvement as the proposed modifications to concessional debt contracts. All welfare changes assume a unit coefficient of partial risk aversion. They increase proportionately for higher values of this coefficient.

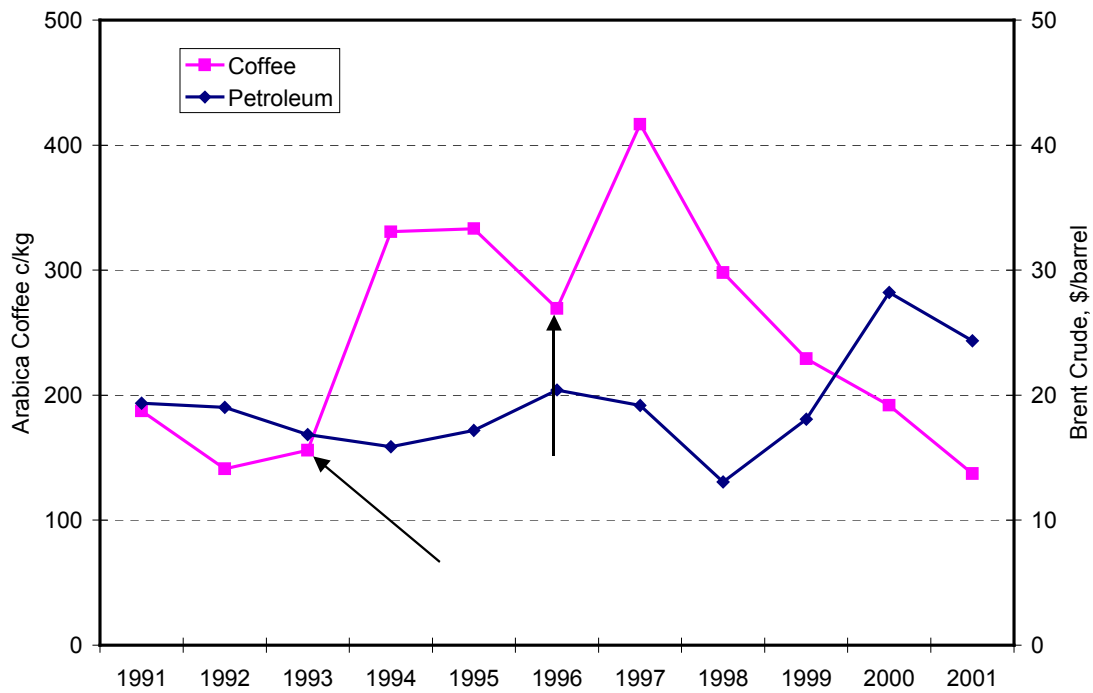
### Country Results: Burundi



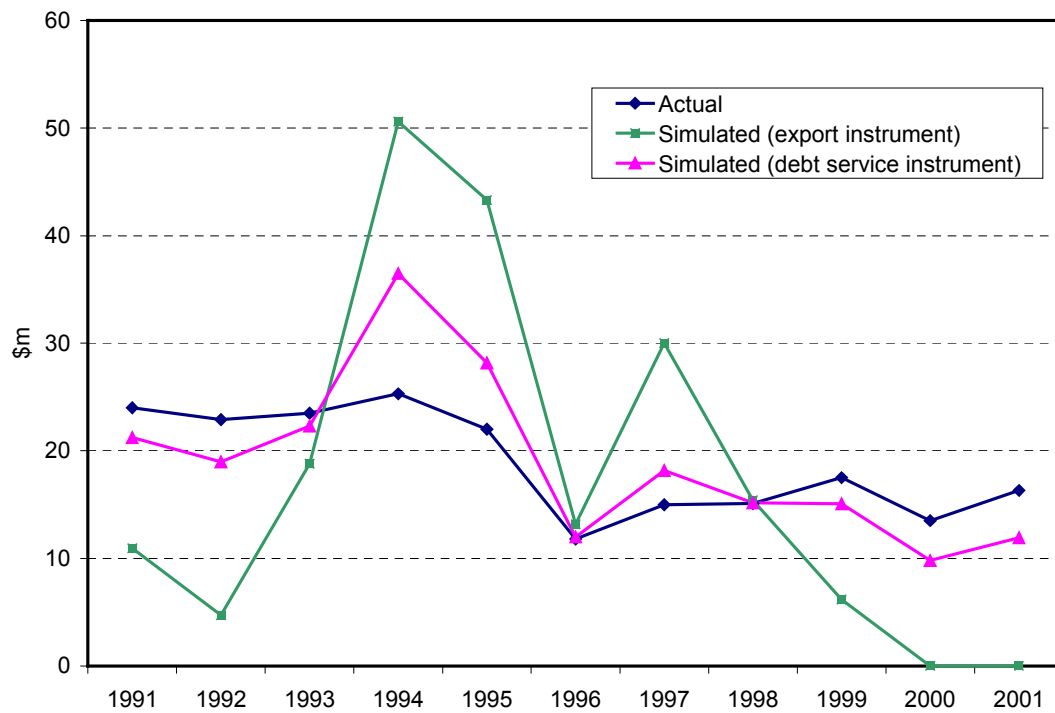
**Figure B1: Burundi - Coffee export revenue and oil import expenditures, 1991-2001**



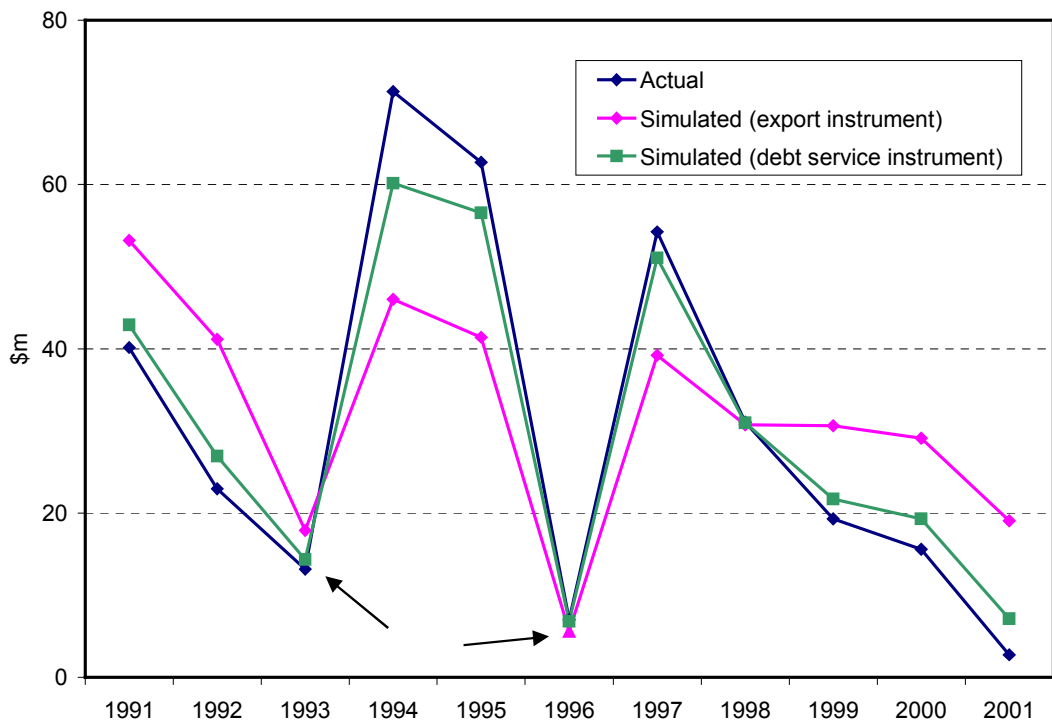
**Figure B2: Burundi - Concessional debt service payments, 1984-2001**



**Figure B3: Burundi - Commodity export and import prices**

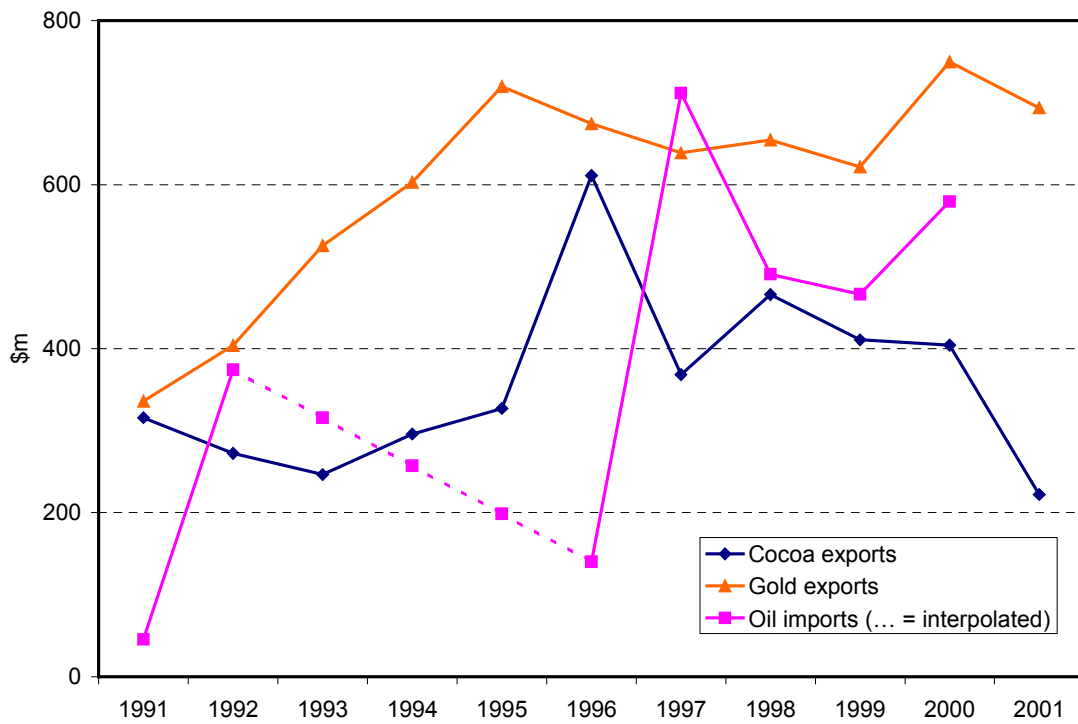


**Figure B4: Burundi – Actual and simulated concessional debt service**

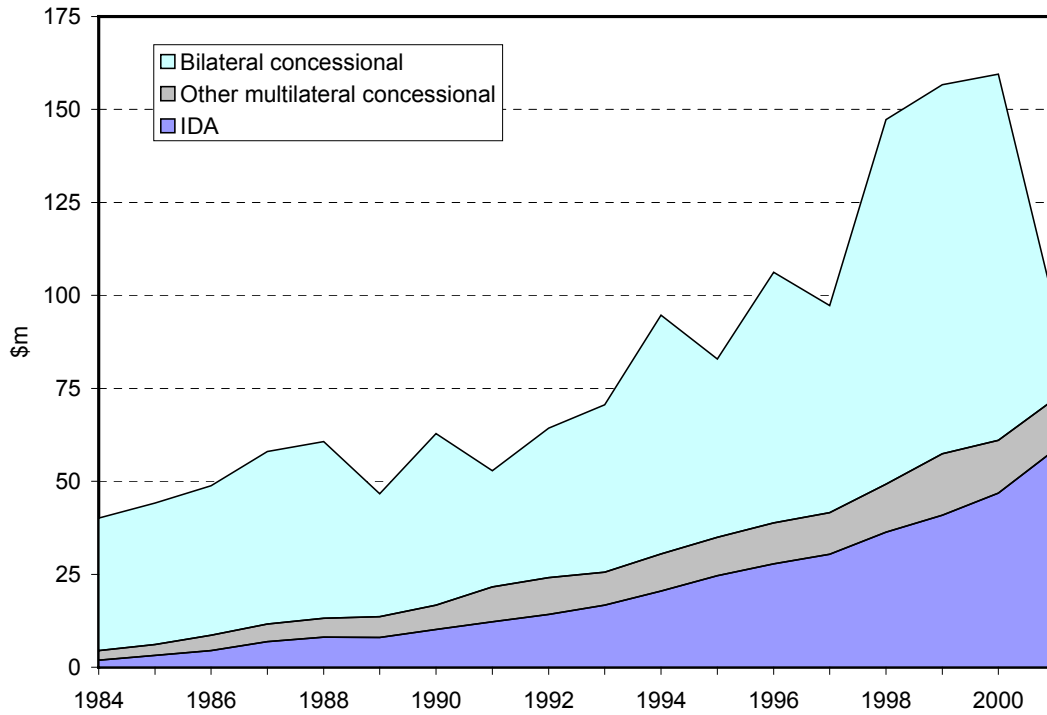


**Figure B5: Burundi – Actual and simulated free foreign exchange**

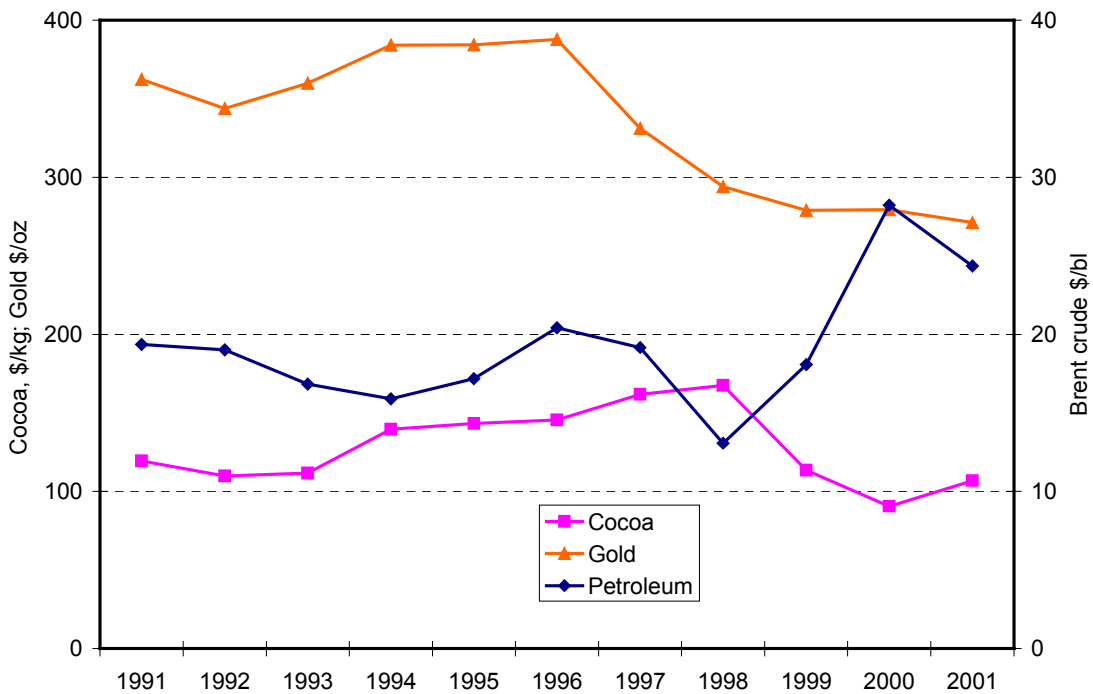
**Country Results: Ghana**



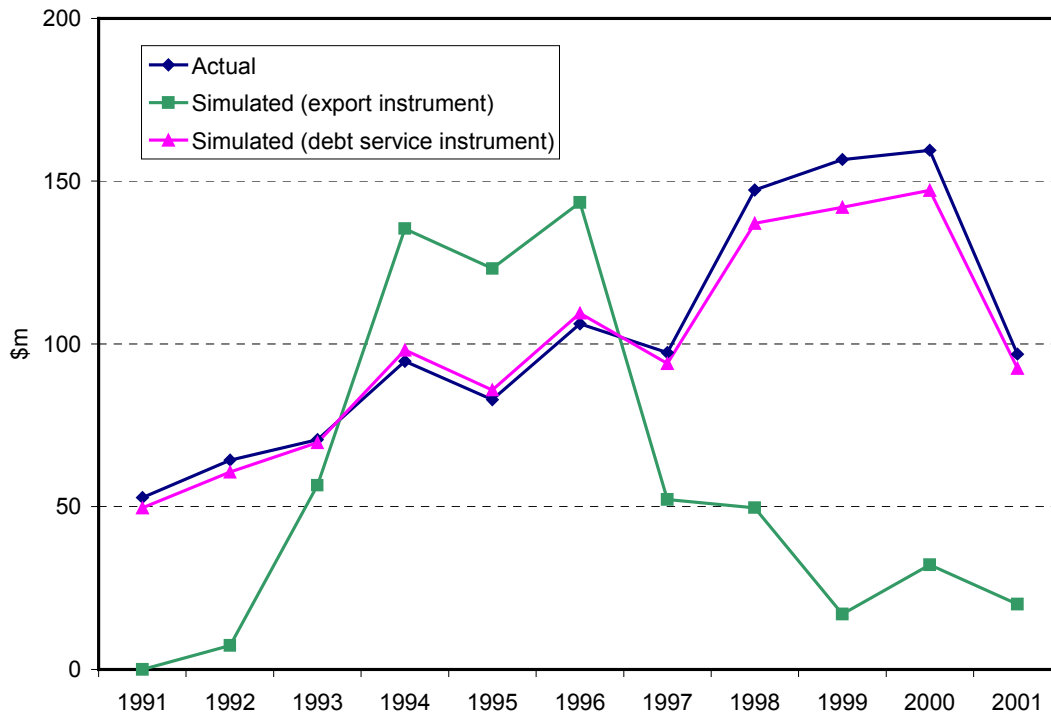
**Figure G1: Ghana – Gold and cocoa export revenue and oil import expenditures, 1991-2001**



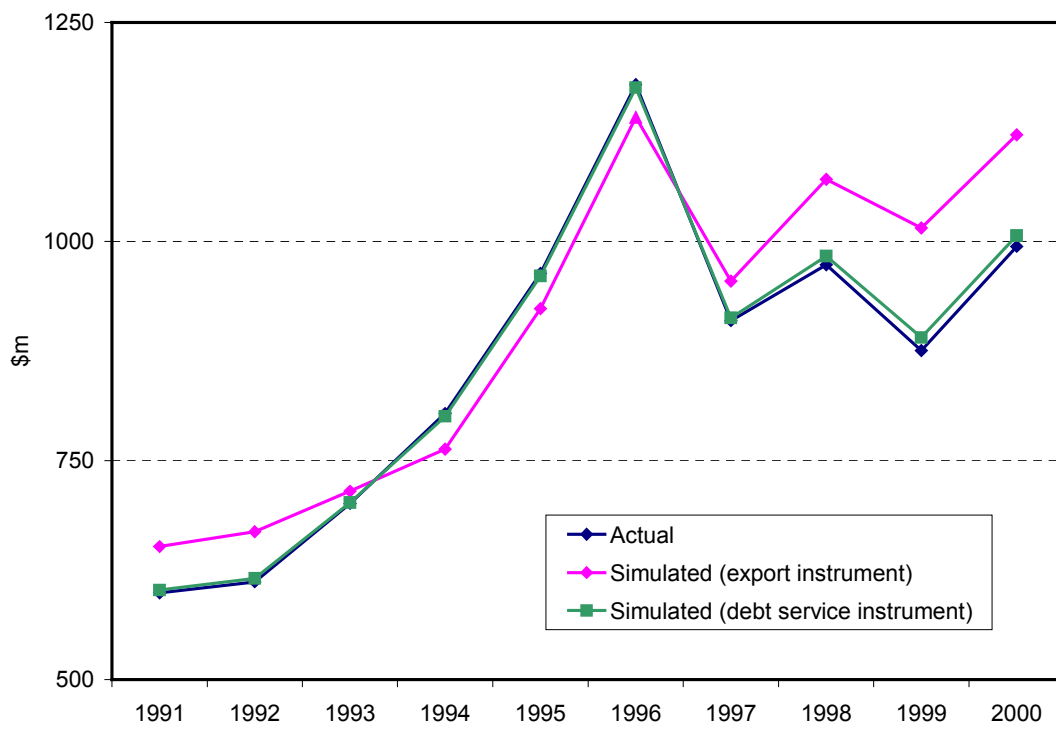
**Figure G2: Ghana - Concessional debt service payments, 1984-2001**



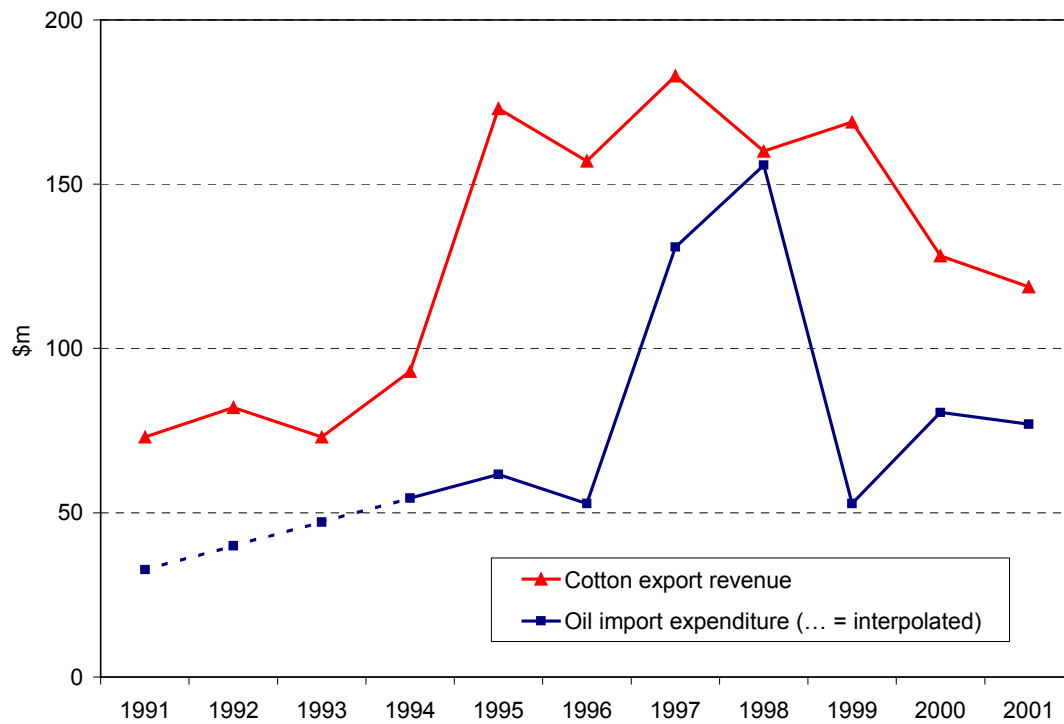
**Figure G3: Ghana - Commodity export and import prices**



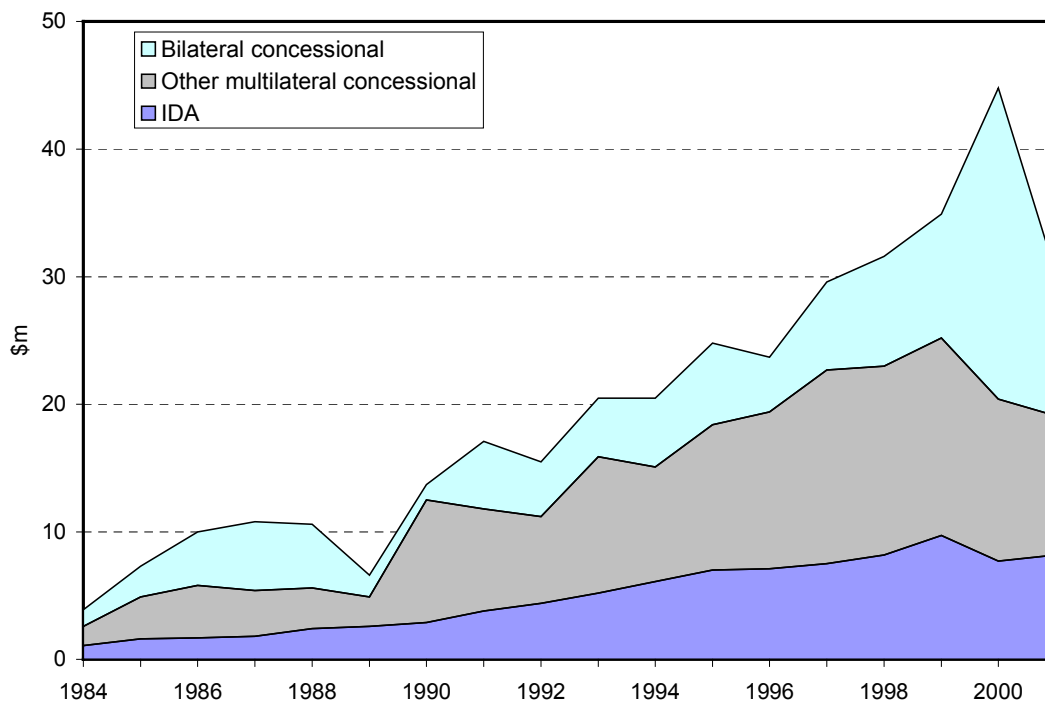
**Figure G4: Ghana – Actual and simulated concessional debt service**



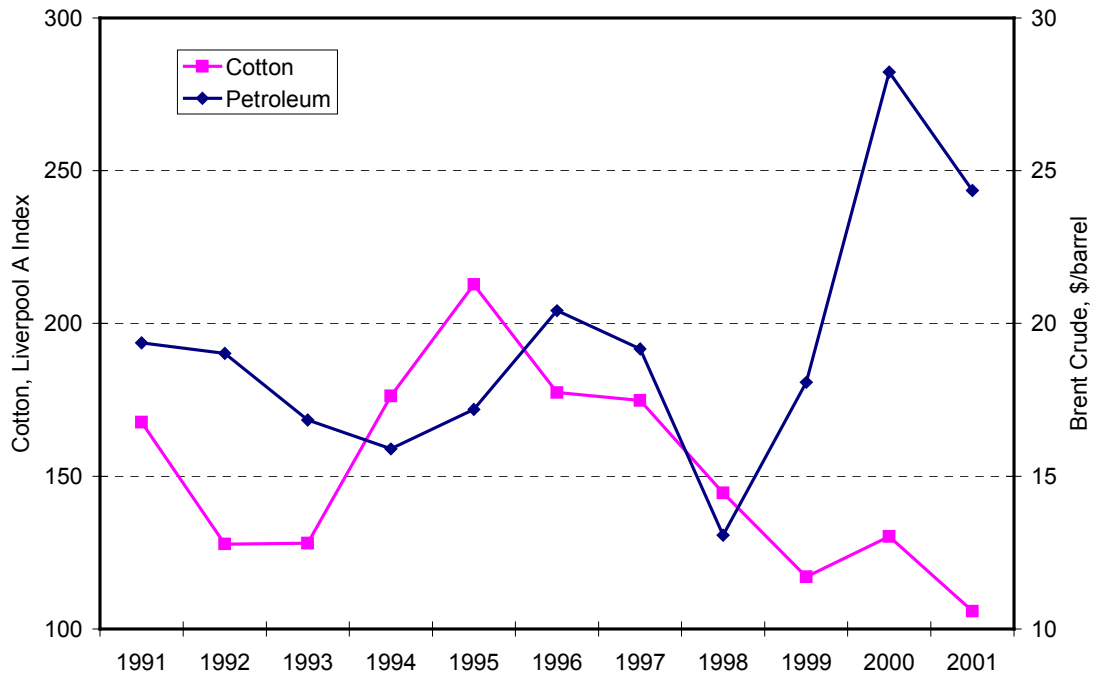
**Figure G5: Ghana – Actual and simulated free foreign exchange**



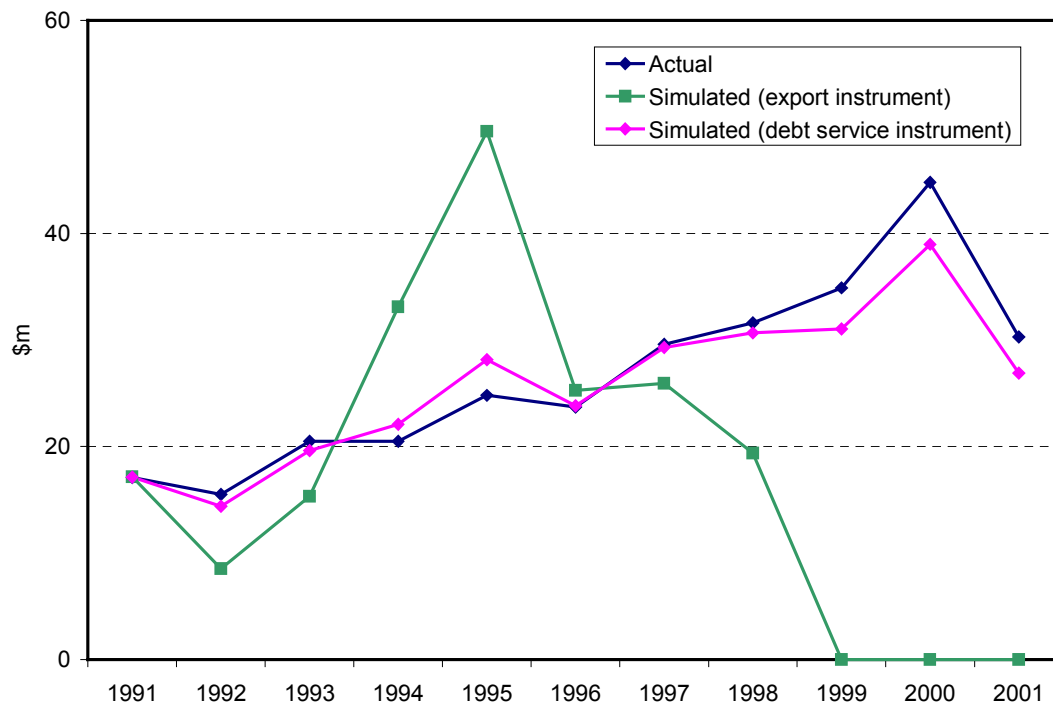
**Figure N1: Benin –cotton export revenue and oil import expenditures, 1991-2001**



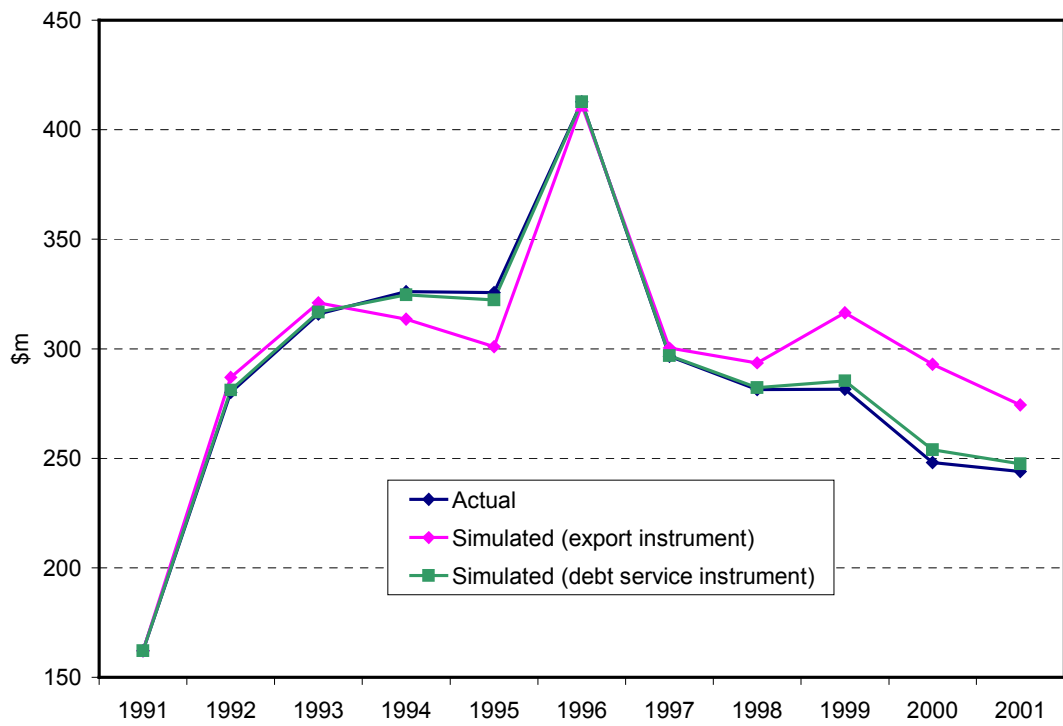
**Figure N2: Benin - Concessional debt service payments, 1984-2001**



**Figure N3: Benin - Commodity export and import prices**



**Figure N4: Benin – Actual and simulated concessional debt service**



**Figure N5: Benin – Actual and simulated free foreign exchange**