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An African Type "MIGA" and Infrastructure-indexed Bonds: Potential Means for Financing Africa's Infrastructure Agenda

by

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ABSTRACT

The paper examines two ways of financing Africa's infrastructure. The first is the potential role of financing Africa's infrastructure projects with bonds indexed to the project. The second is a call for the creation of an African Investment Guarantee Agency (AIGA) to support and strengthen the financing of infrastructure projects. The main objective of AIGA would be to provide non-commercial investment guarantees to African and non-African investors, private or public who are desirous of investing in Africa but do not have the appetite for the above non-commercial risks. Using option-pricing techniques, the author shows that an infrastructure indexed bond is equivalent to a regular bond and a short position on an European put option. The cost of AIGA's guarantees and the associated implied risk premium are also derived. The results of the paper suggests that the value of the infrastructure indexed bond increases monotonically as the value of the project it is financing rises. In addition, the market value of the infrastructure-indexed bonds falls as the value of the project becomes more volatile. The rise in the dividend rate on the project is observed to have an adverse effect on the value of infrastructure-indexed bonds. Lastly, the risk premium embedded in a infrastructure-indexed bond contract is seen as a function of the ratio of the cost of the infrastructure-indexed bond guarantee and the present value of the promised payment.

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1. Introduction

The lack of adequate infrastructure in Africa is perhaps a reason for the continent's economic performance as good and strong infrastructure do support sustainable economic growth and wealth generation. Moreover, in this age of globalization, industries depend more and more on modern and efficient local infrastructure to enhance low operational costs and high quality service.

In Africa and other developing countries, the building and operation of infrastructure is undertaken by governments. However, this trend is changing as the private sector is attempting to get into financing and operation of infrastructure facilities. The demand for private investment has arisen as a result of: (1) the lack of government financial resources; (2) decrease in ODAs for infrastructure projects; and (3) the strong demand for new infrastructure development. Governments of the developing countries are paying attention and are slowly expanding the role of the private sector to implement their infrastructure programmes.

To enhance the participation of the private sector in the financing of infrastructure development projects requires that a sound financing framework be established. Vehicles that could be used to support the financing of projects are the global capital markets. These markets have the depth, maturity, size and the capability of handle complex shocks and therefore have the potential of funding infrastructure projects.

This paper suggests two ways of financing infrastructure programmes in Africa. Fist approach recommended is a call on governments to issue infrastructure bonds. These bonds, it is argued in the paper, could be raised by African countries on the capital markets of the continent. Infrastructure development requires high capital infusion and specific investments, factors that deter the private sector to commit long-term capital. However, by raising the capital required to finance the infrastructure projects from the bond market, financial risks become diversified so as to ensure allocative efficiency.

A second method of financing infrastructure is the setting up of an Africa Investment Guarantee Agency (AIGA). The paper suggests that AIGA objective could be to provide non-commercial investment guarantees to African and non-African investors, private or public who are desirous of investing in Africa but are not prepared to face non-commercial risks.

The paper is organized as follows. Section 2 provides a brief review on the state of Africa's infrastructure. Section 3 provides an overview on the traditional ways of financing infrastructure. Section 4 discusses the pricing of infrastructure bonds. Section 5 argues for the creation of an African Investment Guarantee Agency to support the financing of infrastructure projects. Concluding remarks are made in Section 6.

2. Africa' Infrastructure

In recent years the leadership of the African continent has renewed its interest in achieving full integration of the continent. However, this goal of a united Africa would be very difficult to achieve without physical integration. That this why a key component of the New Partnership for Africa's Development (NEPAD) is devoted to strengthening Africa's weak infrastructure. This section provides a brief overview on the state of the continent's infrastructure and suggestions on how it could be strengthened.

2.1 Transport

Compared to world standards, Africa's infrastructure network is generally very weak. The main mode of transportation is by road, accounting for 90 per cent of urban transport. Yet of the 2 million kilometers of Africa's road networks only about 28 per cent are paved. Moreover, the road density is extremely low, estimated at about 7 kilometres per 100 square kilometres.

The railway connectivity in Africa is also very weak, particularly in central and west Africa. For a continent of size 30 million square kilometers, the rail network accounts for only 89,380 kilometres, translating into 3 kilometres of rail lines per 1,000.

Most of Africa’s international trade depends on maritime transport. Yet a great number of the ports are below international standards. The merchant fleet are also very old, with an average age of 19 years as opposed to a world average of 14 years.

Africa’s air transport networks also needs to be brought to world standards. It is probably easier to fly from an African country to Europe or the US than to fly within Africa. Perhaps the adoption and the implementation of the Yamoussoukro Decision in 2000 could expiate the process of opening up the skies of the continent.

The challenges faced on the continent in the operation of transport include cumbersome customs and administrative procedures, roadblocks, inefficient operators, and inappropriate vehicles, factors contributing to high transportation costs. Transport costs remain very high in Africa, particularly for landlocked countries because of poor infrastructure and weak institutions.

In order to improve the whole transportation network of the continent there is a strong need for the leadership to engage in the promotion and implementation of the infrastructure programmes of the New Partnership for Africa's Development. The short-term action plan proposed by the African Development Bank in 2002 to strengthen infrastructure and services in African countries, to facilitate trade and tourism must be encouraged and immediately implemented. Efforts should also be made by the leadership to construct the missing links of the Trans-African Highway. In addition the facilitation of transport along Africa's major international transport corridors, especially those serving landlocked countries, must be carried out.

The strengthening of the transport system of the continent would not be an easy chore. It would require the total commitment of member countries of the African Union. It would require massive financial support. This calls for a “paradigm shift” from the traditional sources of funds for infrastructure. The leadership may therefore have to turn to other non-traditional methods that would attract private investment in infrastructure. The idea of public-private-partnership projects (PPP) should therefore be embraced by the African leadership.

2.2 Communication

Although there has been some improvements, communication systems in Africa remain very weak. To strengthen regional integration efforts there is the strong need for African countries to modernize existing ICTs. A strengthened ICTs would contribute to growth in trade and financial services as well as reduce the cost of information and linking communities with each other.

In recent years the communications network on the continent has improved significantly, attracting greater investment from local and foreign investors. In most parts of the continent, the growth in fixed line telephone connectivity and mobile telephones services have risen remarkably. Internet use by the population is also up.

Despite the improvements made in this field, compared to the past, more needs to be done if Africa is to bridge the digital divide. It is essential the strengthening of the telecommunication sector of the continent would require the convergence of national policies. This would require: the harmonization of market structures; the creation of conditions that ensure the interconnection among operators in different countries; the harmonization of tariff principles; and reducing the costs of telecommunications services.

2.3 Energy

Africa is endowed with a lot of energy resources — oil, coal, hydroelectricity, natural gas, and biomass and other renewable energy sources. Yet these have not been efficiently tapped or underdeveloped. Those that are developed are often located far from the demand centres. The infrastructure — gas pipelines and electricity transmission and distribution networks — needed to move the supply to consumers remain very weak. To ensure a sufficient, sustainable supply of commercial energy it is important the infrastructure is strengthened.

The regional economic communities are increasing their efforts to promote regional power pools and interconnected electricity grids and master plans for regional power development and environmentally benign power sources. Oil and gas pipelines are also being connected from supply points to consumers. These include the Trans-Mediterranean Gas Pipeline linking Algeria to Italy through Tunisia; the Maghreb and Europe Gas Pipeline linking Algeria to Spain through Morocco; the West African Gas Pipeline supplies Benin, Ghana, and Togo with natural gas from Nigeria; the Mozambique-South Africa Natural Gas Project among others. The regional economic communities continue to promote regional cooperation in the development of hydropower resources. The RECs efforts have contributed to countries jointly developing several hydro power generating facilities.

Although significant progress has been made in strengthening Africa's energy infrastructure, substantial investments are still required in order to bring the infrastructure to world standards. Going forward, there is the strong need for the leadership of the continent to encourage cross-border energy trade, the development of regional power pools, harmonization of regulatory policies and investment codes, and engaging in capacity-building activities.

3. Traditional forms of Financing Infrastructure in Africa[2]

Before arguing that Africa's infrastructure development be financed with infrastructure indexed bonds, it is important that we present a brief discussion on the current form of financing used by African countries. This is important in a number of ways because it contributes to the motivation of the paper. In addition, it allows the recommendation of this paper to be compared with existing methods used by developing, particularly those in Africa, to deal with their infrastructure development needs.

Traditionally, all of the developing economies, including African countries, rely mainly on the World Bank in the financing of infrastructure. The forms of financing offered by the World Bank to the countries include loans, guarantees, equity investments, and political risk insurance. The

2. This section relies heavily on materials on the website of the World Bank (www.worldbank.org). The reader is referred to this site for an in depth discussion on this subject matter.

agencies of the Bank responsible for financially aiding the African countries are: the International Bank for Reconstruction and Development (IBRD) and the International Development Association (IDA), together referred to as the World Bank; the International Finance Corporation (IFC); and the Multilateral Investment Guarantee Agency (MIGA). *What are the functions of these agencies and what contractual arrangements do they reach with developing countries in the development of their infrastructure?*

3.1 Loans and Guarantees by IBRD

The World Bank, through IBRD and IDA, finances both public and private sector projects. IBRD supports middle-income developing countries while the IDA provides assistance to least developed countries. The financial assistance given by the World Bank (IBRD and IDA) is general in the form of loans. However, IBRD has provided limited forms of guarantees to support infrastructure projects undertaken by some of countries. Governments sometimes use the IBRD and IDA loans to finance equity, guarantees, and other forms of financial support. In the next part of this section we provide examples to explain how the World Bank supports infrastructure projects in developing countries.

IBRD charges interests on its loans at a few basis points higher than those charged to AAA-rated bonds. The loans could be contracted in two forms. One form is where, for example, IBRD lends directly to a consortium constructing a super highway and, as required by IBRD's Articles of Agreement, the loan is guaranteed by the host country to IBRD. The second contractual arrangement would be the case where IBRD would lend directly to the host country, which in turn would loan to a public or private company. This contractual arrangement is therefore made up of a loan agreement between IBRD and the host country and a subsidiary loan agreement between the host country and the public or private company.

The guarantees offered by IBRD for private lending takes two forms: a partial risk guarantee protecting lenders against payment defaults arising from breaches of sovereign contractual undertakings to a project, and a partial credit guarantee covering certain debt service payments against all risks, typically for later maturities. IBRD's Articles of Agreement requires that all its guarantees are counter-guaranteed by the host country.

The provision of a partial risk guarantee by IBRD for private lending takes the following structure: (a) loans by commercial lenders; (b) host country contractual undertakings to the public or private company that would support the infrastructure project; (c) an IBRD partial risk guaranteeing loans against private investors debt service defaults arising from host country breaches of contract; and a host country counter-guarantee to IBRD.

To further explain the partial risk guarantee let us take a consortium of public-private-partnership in the construction of a super highway in a developing country and IBRD agrees to provide partial risk guarantees for the project. The host country would then undertake to make compensatory payments to the consortium should there be a specified contractual defaults by the sub-contractors of the project. The IBRD guarantee protects lenders to the consortium against debt service defaults resulting from the host country's breach of this obligation. The host country in this case should have a counter-guarantee to IBRD.

Alternatively, IBRD could provide a partial credit guarantee to the commercial lenders to protect them against default by the consortium on debt service payments. This structure would involve the same basic agreements as the partial risk guarantee, but would not require contractual undertakings from the host country to the consortium.

3.2 IBRD financing in IDA countries

Generally, IBRD does not finance projects in poorer developing countries. This is because these countries are not in a position to pay IBRD's market lending rates as their cridt worthiness is very low. These countries receive only concessionary IDA lending (referred to as IDA-only countries).

In exceptional cases, IBRD has offered loans for certain projects in IDA-only countries that generate substantial foreign exchange revenues (referred to as enclave projects). In addition, these projects typically include an offshore escrow account for debt service payments and a guarantee to IBRD from a creditworthy third party, such as the project's private shareholders. Enclave projects could also be eligible for an IBRD guarantee.

3.3 IDA Loans

IDA offers fairly favourable loans to poor developing nations. These loans are made generally on very good concessional terms, with maturities of thirty-five to forty years and a 0.75 percent interest rate. IDA loans to a host country, which in turn lends the funds to the company responsible for developing the infrastructure project. In the example used in the previous section, IDA would support the development of the super highway by entering into a loan agreement with the host country. The host country would then loan the funds to the Consortium backed by a project agreement between the Consortium and IDA.

3.4 Equity Financing

IBRD and IDA do not invest directly in infastructure projects of a developing countries. In other words either of the two agencies hold equity stakes in projects carried out in the countries they loan to. However, either of the agencies could provide loans or credits to a country to finance its equity investment in an infrastructure project. In the example being used in this Section, the host country could obtain an IBRD loan or an IDA credit to finance an equity stake in the Consortium developing the super highway. In this case, IBRD would have a loan agreement with the host country. The host country would then have a subsidiary loan agreement with a parastatal company which would then use the funds to have an equity stake in the Consortium.

3.5 Financing guarantees and debt refinancing

IBRD and IDA also provides loans to countries to finance guarantees offered by host countries to investors who are undertaking infrastructure development. For example, a country could finance a loan guarantee issued by an independent guarantor mandated by the government with a loan from IBRD or IDA. The structure of such an arrangement would involve IBRD or IDA providing a loan to the host country. The host country would then use this funds, through a parastatal agency such a financial intermediary, to provide guarantees to commercial lenders (banks) who have loan agreements with the project developers (the Consortium in our example).

3.6 Financial intermediaries, investment funds, and facilities

In some cases IBRD and IDA do lend to a host country to finance financial intermediaries or an investment fund or other facility that would provide loans, equity, guarantees, take-out financing, or other financial support for special projects. An example is where IBRD provides loan to a host country to finance a government-sponsored fund that lends to selected private sector infrastructure projects.

3.7 IFC's loans, loan syndication, equity, and quasi equity

The IFC generally offers loans or holds equity and quasi equity investments in private ventures only. Unlike the World Bank, IFC financing requires no direct government guarantees of repayment. Loans made by the IFC attract market interest rates. To hedge against currency transfer and other political risks to lenders, the IFC provides an extensive loan syndication program (known as B-loans) by extending its lender-of-record umbrella to participating banks. B-loans are always coupled with loans funded from IFC's own resources (A-loans).

In the construction of the supper highway example, IFC could have an equity stake in the Consortium with an A- loan and a syndicated B-loan. At the same time it could choose to have participation agreements with syndicated commercial lenders who are providing loans to the Consortium.

3.8 MIGA’s political risk insurance

MIGA provides guarantee for foreign equity and related debt investments to cover unforeseen political risk. The coverage are for war and civil disturbance, expropriation, and currency transfer risks. MIGA also offers guarantees for breach of contracts which the host country prevent claimants to seek relief through judicial courts. In our example, MIGA insurance would be provided to the Consortium developing the super highway.

3.9 Group support

The World Bank also provides financial assistance to infrastructure projects supported by all its agencies so as to encourage foreign direct investment (FDIs). In the example of the construction of the highways, the Consortium could receive support from the World Bank through IFC equity (A- and B-loans), MIGA political risk insurance, and an IBRD partial risk guarantee.

4. The Pricing of the Infrastructure-indexed Bonds

Section 3 has provided a brief discussion on World Bank financial assisted programmes for developing countries. One of the aims of this paper is to suggest that African countries consider issuing bonds indexed to infrastructural projects as a way to complementing the sources of funds allocated to infrastructure projects. This section proposes a method of pricing such bonds.

Option pricing techniques are very flexible such that they can easily be used in the valuation of most financial instruments. In this section, we use the technique to suggest a pricing formula for the infrastructure bond. It also assumed under the contractual framework that the bond-holders would take over the project if the borrower defaults on the payment. Although the future value of the project is not known, we assume that all parties have full knowledge of the initial market value of the project. Under this assumption, the borrower will rationally default at any time during the contract period if the market value of the project is less than the outstanding balance of the infrastructure-indexed bond.

To finance the infrastructure project in some part of an African country or in a region of the continent, we assume that the government of the country in question or a regional economic community would invite private sector participation in the financing of the project through the issue of bonds linked to the project. Let us assume that the face value of the Bond is *F* and the market of value of the project is *V*, then the expected value of the infrastructure-indexed bond at the end of the contract is:

$$\mathrm{Min}[F, V]. \tag{1}$$

We make a further assumption that embedded in *F* are all the necessary costs incurred by the bond-holders upon defaulting of the contract.[3]

In addition to the above assumption, we shall make the usual assumptions for modelling continuous-time asset-pricing models: (i) assets are traded in a frictionless or perfect market, where there are no taxes, transactions costs, or short sale restrictions, and all assets are perfectly divisible; (ii) trading of assets is done continuously; and (iii) the value of the project follows a continuous-time diffusion process. For reasons of parsimony, the interest rates are also assumed to be deterministic.

Lastly, we assume that lenders and borrowers have the same information on, and identical beliefs in, the prospects for the project. Both therefore agree on the diffusion process followed by the value of the project. Let *V(t)* be the market value of the project and its stochastic process be of the form:

$$\frac{dV}{V} = \left(\alpha_v - \frac{\delta}{V}\right)dt + \sigma_v dz_v \ , \tag{2}$$

3. These costs may include bankruptcy, legal, and reputation costs.

where dz_v is a standard Brownian motion, with mean zero and variance dt. In equation (2), it is assumed that the project pays out dividends at a constant rate, δ. The diffusion part (the second part on the right-hand side) of equation (2) makes the instantaneous rate of appreciation of the project uncertain. Hence, the expected rate of appreciation of the project is $(\alpha_q - \delta / Q)$.

4.1 Valuation of the infrastructure indexed bond

Since the bond is indexed to the market value of the project then the market value of the bond at any given time is $B(V, t)$. Using the Ito's lemma, the drift and the diffusion of the bond could be expressed as:

$$dB = B_v dV + \frac{1}{2} B_{vv} (dV)^2 + B_t dt \quad , \tag{3}$$

which, upon simplifying, yields:

$$\frac{dB}{B} = \alpha_b dt + \sigma_b dz_v \, , \tag{4}$$

where

$$\alpha_b = \left[\frac{1}{2} \sigma_v^2 V^2 B_{vv} + \left(\alpha_v - \frac{\delta}{V} \right) V B_v + B_t + c \right] / B \quad , \tag{5}$$

$$\sigma_b = \frac{\sigma_v V B_v}{B} \, . \tag{6}$$

and c is the instantaneous coupon rate paid to bond holders.

Proposition 1: *Based on the assumptions and the framework proposed the partial differential equation governing the infrastructure-indexed bond, which pays an instantaneous coupon rate of c is*

$$\frac{1}{2}\sigma_v^2 V^2 B_{vv} + (rV - \delta)B_v + B_t - rB + c = 0 \ . \tag{7}$$

Proof:

Standard arbitrage arguments common in the options-pricing literature are employed for the proof. Consider a portfolio, *H*, made up of a share ω in the project and (1 - ω) in the bond. The instantaneous return on such a portfolio *dH*, is:

$$dH = \omega(\alpha_v dt + \sigma_v dz_v) + (1 - \omega)[\alpha_b dt + \sigma_b dz_v] \qquad . \tag{8}$$

Rearranging,

$$dH = (\omega\alpha_v + (1 - \omega)\alpha_b)dt + [\omega\sigma_v + (1 - \omega)\sigma_v]dz_v \ . \tag{9}$$

H would be a riskless portfolio if ω is chosen such that

$$\omega\sigma_v + (1 - \omega)\sigma_b = 0 \ , \tag{10}$$

or

$$\omega = \frac{\sigma_b}{\sigma_b - \sigma_v} \ . \tag{11}$$

An investor who holds $\sigma_b/(\sigma_b - \sigma_v)$ units in the value of the project and $-\sigma_v/(\sigma_b - \sigma_v)$ in the bond would earn the riskless rate of return *r* if there are no arbitrage profits to be made. Hence:

$$dH = r\left[\frac{\sigma_b}{\sigma_b - \sigma_v} - \frac{\sigma_v}{\sigma_b - \sigma_v}\right]dt \ . \tag{12}$$

Equating equation (12) with the drift term of equation (9), with the substitution of ω, we have:

$$\left[\left(\frac{\sigma_b}{\sigma_b - \sigma_v}\right)\alpha_v - \left(\frac{\sigma_v}{\sigma_b - \sigma_v}\right)\alpha_b\right] = r\left[\frac{\sigma_b}{\sigma_b - \sigma_v} - \frac{\sigma_v}{\sigma_b - \sigma_v}\right] . \tag{13}$$

Equation (7), the partial differential equation governing the value of the bond is obtained by the substitution of equations (5) and (6) into equation (13) and rearranging.

An intuitive explanation in the finance literature for the differential equation that summarizes the bond pricing is the following. The first term on the left-hand side of equation (7) captures Jensen's inequality effect coming from the variance of the value of the project. The second term represents the risk-adjusted expected drift of the value of the project. The third term reflects the shrinking time-to-maturity. The last term represents the net flows to the lender.

Proposition 2: *The present value of the bond under the set of assumptions is:*

$$B(V, \tau) = \frac{c}{r}(1 - e^{-r\tau}) + Fe^{-r\tau} - P(V, F, \tau) , \tag{14}$$

where $P(V, F, \tau)$ *is a European put option on the project,* V*, with constant dividend* δV *and an exercise price equal to the principal payment,* F*. The value of the put option is*

$$P(V, F, \tau) = Fe^{-r\tau}N(d_1) - Ve^{-\delta\tau}N(d_2) , \tag{15}$$

where

$$d_1 = \frac{\log(F/V) + \left((\delta - r) + \frac{1}{2}\sigma_v^2\right)\tau}{\sigma_v\sqrt{\tau}} ,$$

$$d_2 = d_1 - \sigma_v\sqrt{\tau} ,$$

and N(.) is the cumulative normal distribution function.

Proof:

Rewrite equation (1) as:

$$\text{Min}[F, V] = F - \max[0, F\text{-}V]. \quad (16)$$

What equation (16) says is that the value of the infrastructure-indexed bond is equivalent to a regular bond (with face value of *F*) and a short position on a European put option on the project, with the time left to maturity τ (*T-t*) and the strike price equal to the final payment of the infrastructure-indexed bond, which by our assumption is the principal, *F*. The first term of equation (14) is the present value of the stream of coupon payments that would accrue to the bond holder over the life of the bond. The second term is the present value of the face value of the bond. The the put option value follows from Atta-Mensah (1992), Merton (1973), and Geske (1979). Lastly, by performing the relevant differentiations, we find that equations (14) and (15) correspond to equation (7), the stochastic partial differential.

Propositions 1 and 2 indicate that the value of the infrastructure-indexed bond is a function of the value of the project, creating an incentive needed to ensure that the project is carried through. In the next section we perform simple comparative statics to examine how the factors underlying the pricing formula impact on the infrastructure indexed bonds.

4.2 Properties of the infrastructure-indexed bonds

This section focuses on assessing factors that impact on the value of the indexed-bonds. This is carried out by studying the influence of key parameters on the pricing formula derived in the last section for the infrastructure indexed bonds.

Proposition 3: *The value of the infrastructure indexed bond increases monotonically as the value of the project it is financing rises.*

Proof:

Differentiating equation (14) with respect to *Q* and using equation (A10) in Appendix A:

$$\frac{\partial B}{\partial V} = -\frac{\partial P}{\partial V} = -(-e^{-\delta\tau}N(d_2)) \geq 0 \tag{17}$$

Remarks: The result corroborates conventional wisdom, which suggests that the higher the value of the project, the greater the size of the infrastructure-indexed bond, given the same level of project risk. An explanation is that as the market value of the project that the indexed bond is being used to finance increases, the probability that the put call embedded in the bond finishes out-of-money also increases, contributing to the rise in the value of the bond. Vice-versa the fall in the market value of the project reduces the value of the indexed bond, reducing the debt burden of the borrower. This result is very important because unlike the traditional bond whose value remains the same regardless of the value of the project, in the case of the indexed bonds its value is directly linked to the market value of the project.

Proposition 4: *The market value of the infrastructure-indexed bonds falls as the value of the project becomes more volatile.*

Proof:

Differentiating equation (14) with respect to the variance of *V*, σ_v, and using equation (A16) in Appendix A:

$$\frac{\partial B}{\partial \sigma_v} = -\frac{\partial P}{\partial \sigma_v} = -(\sqrt{\tau}Ve^{-\delta\tau}N'(d_2)) \leq 0 \ . \tag{18}$$

Remarks: The result shows that risk-averse investors would be very reluctant to hold the infrastructure-indexed bonds if the project is very risky (measured by the volatility of the market value of the project). In a sense this imposes discipline on borrowers and prevents them from taking up risky ventures as investors are less likely to “bank roll” them. Technically, the rise in the volatility of the market value of the project increases the value of the put option embedded in the indexed bond. This is because a put call has no downside risk, since the value of the put is zero irrespective of how far it finishes out of the money. What the increase in the volatility of the market value of project does is to increase the likelihood that the put option would finish in the money.

Proposition 5: *The value of the infrastructure-indexed bonds rises monotonically with the face value, F, of the bond.*

Proof:

Differentiating equation (14) with respect to *F* and using equation (A21) in Appendix A:

$$\frac{\partial B}{\partial F} = e^{-r\tau}(1 - N(d_1)) \geq 0 \ , \tag{19}$$

since $0 \leq N(d_1) \leq 1$.

Remarks: To explain the results one needs to recall that technically the indexed bond could be decomposed into a regular bond and short position on a put option on the project with a strike equal to the face value of the regular bond. Hence an increase in the principal of the indexed bonds leads to an increase in the value of the regular bond component of the indexed bond and at the same time an increase in the short position of the put option. On net, the indexed bond rises in value because the regular bond dominates the put option. Intuitively, the results means that investors would be attracted to the indexed bonds if its face value should rise.

Proposition 6: *A tightening of monetary policy has an negative impact on the market value of an infrastructure-indexed bond.*

Proof:

Differentiating equation (14) with respect to the interest rate, *r*, and using equation (A24):

$$\frac{\partial B}{\partial r} = -\frac{c}{r^2}[1-(1+r\tau)e^{-r\tau}] - \tau F e^{-r\tau}(1-N(d_1)) \leq 0 \ , \tag{20}$$

since $0 \leq N(d_1) \leq 1$ and given that r and τ are non-negative we should expect $[1-(1+r\tau)e^{-r\tau}] \geq 0$

Remarks: Intuitively, interest rates bear a negative relationship with the price of a financial asset. That is why asset prices fall with the tightening of monetary policy. The result shows that the increase in the interest rates leads to a decline in the market value of infrastructure-indexed bonds. The results can be explained by the fact that the rise in interest rates reduces the present values of the bond component of the infrastructure-indexed bond and the exercise price of the put option. These factors combine to depress the market value of the infrastructure-indexed bond.

Proposition 7: *The rise in the dividend rate on the project has an adverse effect on the value of infrastructure-indexed bonds.*

Proof:

Differentiating equation (14) with respect to the dividend rate, δ, and using equation (A26):

$$\frac{\partial B}{\partial \delta} = -\tau V e^{-\delta\tau} N(d_2) \leq 0 \ . \tag{21}$$

Remarks: An intuitive explanation of this result is that the rise in the dividend make ownership in the project more attractive than bearing the indexed bonds as a result the fall in the value of the indexed bonds. A technical explanation is that the rise in the dividend rate increases the value of the put option component embedded in the index bonds. Since the contribution of the put option to the value of the indexed bond is negative a rise in the dividend rate on the value of the indexed bond is negative.

Proposition 8: *The term-to-maturity date has an ambiguous effect on the value of infrastructure-indexed bonds.*

Proof:

Differentiating equation (14) with respect to the term left to maturity, τ, and using equation (A27):

$$\frac{\partial B}{\partial \tau} = ce^{-r\tau} - rFe^{-r\tau}(1 - N(d_1)) - V\delta e^{-\delta\tau}N(d_2) + \frac{\sigma_v}{2\sqrt{\tau}}Fe^{-r\tau}N'(d_1) \quad . \tag{22}$$

Clearly, the sign for equation (22) is indeterminate.

Remarks: This result demonstrates that investors are indifferent between holding short- and long-term infrastructure-indexed bonds.

5. An African Investment Guarantee Agency

Another way of strengthening the financing of Africa's infrastructure is through the establishment of an African Investment Guarantee Agency (AIGA). AIGA's function could be similar to the World Bank's MIGA but would cater to only investments in Africa, particularly in the area of infrastructure.

MIGA, which was created in 1988, aims at promoting FDIs in developing countries by providing guarantees to investors against non-commercial risks such currency transfers restrictions, expropriation and war and civil disturbance. Since the establishment of MIGA it has provided a cumulative guarantees totalling $13.5 billion, within guarantees in 2004 adding up to $1.1 billion. The membership of MIGA, which is opened to all the member countries of the World Bank, currently stands at 164. In addition to its investment guarantees, MIGA also offers technical assistance to its members in strategies that contribute to the promotion of FDIs. Its legal wing could be used is settling disputes and disagreements between investors and governments.

5.1 Suggested Framework for AIGA

AIGA could have a mission of promoting investments in Africa, particularly in the area of infrastructure development. *Why AIGA and not MIGA?* Despite the its good work in facilitating and promoting FDIs to developing countries, MIGAs focus is very broad and therefore the assistance to Africa could be smaller than what is expected. AIGA, if properly created, would focus only on Africa as its members would be made up of only African countries and charged with the sole task of assisting public or private investments on the continent. This agency is also needed because of the difficulty of African countries face in attracting productive capital for financing projects, as international investors perceive the countries to be of high risk due to political instability and other non-financial risks that have plagued the continent for some time now. Moreover, if it is managed well, AIGA has a potential of generating enough profits that could be ploughed back into the continent's economy.

The main objective of AIGA would be to provide non-commercial investment guarantees to African and non-African investors, private or public who are desirous of investing in Africa but do not have the appetite for the above non-commercial risks. Like those covered by MIGA, these risks could include an investors inability to repatriate funds because of an unstable political environment in a country, war or civil or sectarian conflicts and unlawful nationalization, confiscation and seizure of the investment projects by the government of a host country. The exact definition of risks could be left in the hands of a legal team that would be responsible for carving out the legal text and protocols that would govern the operation of the Agency.

The structure of AIGA should be such that its membership is opened mainly to African countries. However, a small fraction of ownership could go to other countries. Each member would contribute to the initial capital needed to operate the agency plus an agreed annual membership fee. The membership of the Agency would have to agree on the currency of operation. A suggestion is that the currency be convertible in international financial markets. Depending on the financial health of the Agency members could be paid dividends from accrued profits.

The Board of Directors of AIGA could be made up of the Finance Ministers of the African countries who hold membership in the Agency. Representatives of non-African countries who are members could be non-voting members of the board. The Board would be responsible for appointing the President of the Agency. Depending on the structure of the Agency, the Board could appoint a number of vice-presidents. The Board should be empowered to set policies, regulations and rules that would ensure the smooth operation of the Agency. The President is empowered to oversee the day-to-day activity of the Agency.

Under the direction of the Board, the President would approve all guarantee-contracts between the Agency and investors. However, the President must ensure that not all investors receive guarantees. For example, investment that are very risky and have high chance of failure should not be covered so as to reduce “moral hazards” or “adverse selection” bias. In other words the Agency, led by the President, should strike a balance between ensuring the sound financial health of the Agency and the provision of coverage that would attract productive investment to the continent.

In addition to a number of units that must be created to assist the President in running the Agency, AIGA should have a is risk management unit. The risk management unit would be responsible for managing all the risk exposures the Agency faces. The unit must be equipped with modern tools, techniques and the best minds in the field to effectively manage all forms of risk. As part of its assignment the unit must be able to determine the actuarial fair premiums each applicant must pay in order to receive a guarantee from AIGA. The unit would in turn invest the premiums on behalf of AIGA in both financial and physical assets. It is very important the President, under the direction of the Board, would have to keep a careful eye on the risk management unit.

5.2 The cost of guarantee

Let us consider a case in which there is a third party that guarantees to pay the lender should the borrower default.[4] This contract, between the borrower and the guarantor, would require that the borrower surrender the project to the guarantor in the event of a infrastructure-indexed bond default. Note that the project could be the assets of the borrower or the value of the project being financed with the infrastructure-indexed bond.

Before we provide the pricing formula for the cost of the guarantee, we analyze the pay-offs under various states of the world. On the maturity date of the infrastructure-indexed bond, if the value of the project, Q, exceeds the promised payment of the infrastructure-indexed bond, F, then the borrower pays the lender, F, and keeps $Q - F$. On the other hand, if the value of the project, Q, is less than the promised payment of the infrastructure-indexed bond, F, the third party pays the lender F and takes a loss of $F - Q$, with the borrower receiving nothing.

Proposition 10: *The cost of the infrastructure-indexed bond guarantee is equivalent to a European put option, the underlying asset of which is Q and the exercise price F.*

Proof:

The contractual arrangement suggests that, at the maturity date, the lender will receive the promised payment, F, regardless of the state of the world. Thus, to the lender the infrastructure-indexed bond is riskless. The net receipt for the borrower is $max(0, Q - F)$, with or without a guarantee. The net receipt to the guarantor is $min(0, Q - F)$, which is non-positive. As a result of the guarantee, the borrower receives an additional cash inflow of $-min(0, Q - F)$, or $max(0, F - Q)$. Hence, if $G(\tau)$ is the value of the guarantee to the borrower, then

$$G(0) = max(0, F - Q), \tag{23}$$

which is equivalent to a put option. Equation (15) gives the exact formula for evaluating the option.

4. The discussion in this section is based on Merton (1977).

5.3 Risk premium

In this section, we attempt to derive an expression for the risk premium embedded in the infrastructure-indexed bond contract. To a lender, a infrastructure-indexed bond that is guaranteed by the borrower is riskless. The difference between the yield on a infrastructure-indexed bond that is not guaranteed and one that is guaranteed is a measure of the risk premium.

Proposition 11: The risk premium embedded in a infrastructure-indexed bond contract is a function of the ratio of the cost of the infrastructure-indexed bond guarantee and the present value of the promised payment.

Proof:

Let $y(\tau)$ be the implied yield of the debt, F, when there is no guarantee, implying that the present value of the debt is $F\exp(-y(\tau)\tau)$. With the infrastructure-indexed bond guaranteed, the lender is assured F, which is either paid by the borrower or the guarantor. The present value of the guaranteed infrastructure-indexed bond is $F\exp(-r(\tau)\tau)$, where $r(\tau)$ is the riskless rate of return. In the absence of arbitrage opportunities, we should expect:

$$G(\tau) + Fe^{-y\tau} = Fe^{-r\tau} , \tag{24}$$

from which the implied risk premium is derived as:

$$\Pi = y - r = -\frac{1}{\tau}\log\left(\left[1 - \frac{G(\tau)}{Fe^{-r\tau}}\right]\right) . \tag{25}$$

Equation (26) gives an expression for the implied risk premium.

5.3.1 Factors influencing the risk premium

In section 3, we showed that the value of the project plays an important part in the determination of a infrastructure-indexed bond contract. In this section, we examine factors that influence the risk premium.

Proposition 12: *The risk premium is negatively correlated with the value of the project.*

Proof:

Differentiate equation (26) with respect to Q:

$$\frac{\partial \Pi}{\partial Q} = -\frac{1}{\tau}\left(\frac{e^{-\delta\tau}}{Fe^{-r\tau} - G(\tau)}\right)N(d_2) \le 0 \ , \qquad (26)$$

since $N(d_2) \ge 0$ and $G(\tau) < Fe^{-r\tau}$.

Remarks: An explanation for this result is that an appreciation in the value of the project leads to a lesser chance of the borrower defaulting on the promised payment of the infrastructure-indexed bond. The risk premium falls, reflecting the reduced risk of default.

Proposition 13: *The risk premium rises as the value of the project becomes more volatile.*

Proof:

Differentiate equation (26) with respect to σ_q:

$$\frac{\partial \Pi}{\partial \sigma_q} = \frac{1}{\sqrt{\tau}}\left(\frac{Qe^{-\delta\tau}}{Fe^{-r\tau} - G(\tau)}\right)N'(d_2) \ge 0 \ , \qquad (27)$$

since $N'(d_2) \ge 0$ and $G(\tau) < Fe^{-r\tau}$.

Remarks: An increase in the volatility of the value of the project increases the option-value component of the infrastructure-indexed bond. Consequently, the risk of default by the borrower rises, resulting in the rise in the risk premium.

Proposition 14: *The impact of monetary policy on the risk premium is indeterminate.*

Proof:

Differentiate equation (26) with respect to *r*:

$$\frac{\partial \Pi}{\partial r} = \frac{1}{\tau}\left(\frac{Fe^{-r\tau}}{Fe^{-r\tau} - G(\tau)}\right)\left(\frac{G(\tau)}{Fe^{-r\tau}} - N(d_1)\right) . \tag{28}$$

Remarks: The results indicate that the impact of monetary policy on the risk premium is indeterminate. A common view held by market analysts is that an expansionary monetary policy will help reduce risky spreads. Our result, however, indicates that the impact of monetary policy on the risky spread cannot be ascertained.

6. Conclusion

In this paper, we have examined the potential role of financing Africa's infrastructure projects with bonds indexed to the project. Developing a nation's infrastructure requires massive capital infusion, which the private sector are reluctant to commit to in the long run. However, capital markets have the depth, maturity, size and the capability of handle complex shocks and therefore have the potential of funding infrastructure projects. Hence, a nation could rely on the bond market to raise the capital it needs to finance infrastructure projects. This would ensure diversification of financial risks and allocative efficiency.

The paper also calls for the creation of an African Investment Guarantee Agency (AIGA) to support and strengthen the financing of infrastructure projects. The proposes that AIGA's function could be similar to the World Bank's MIGA but would cater to only investments in Africa, particularly in the area of infrastructure. The main objective of AIGA would be to provide non-commercial investment guarantees to African and non-African investors, private or public who are desirous of investing in Africa but do not have the appetite for the above non-commercial risks.

Using option-pricing techniques, the paper shows that an infrastructure indexed bond is equivalent to a regular bond and a short position on an European put option. We also derive the cost of AIGA's guarantees and the associated implied risk premium. The results of the paper indicates that the value of the infrastructure indexed bond increases monotonically as the value of the project it is financing rise. Moreover, the market value of the infrastructure-indexed bonds falls as the value of the project becomes more volatile. The rise in the dividend rate on the project is observed to have an adverse effect on the value of infrastructure-indexed bonds. Lastly, the risk premium embedded in a infrastructure-indexed bond contract is seen as a function of the ratio of the cost of the infrastructure-indexed bond guarantee and the present value of the promised payment.

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Appendix A: Properties of the Put Option

A1. The value of the put option

In the text, an expression for valuing a European put option was given as:

$$P(V, F, \tau) = Fe^{-r\tau}N(d_1) - Qe^{-\delta\tau}N(d_2) \ , \tag{A1}$$

where

$$d_1 = \frac{\log(F/V) + \left((\delta - r) + \frac{1}{2}\sigma_v^2\right)\tau}{\sigma_v\sqrt{\tau}} \ , \tag{A2}$$

$$d_2 = d_1 - \sigma_v\sqrt{\tau} \, , \tag{A3}$$

and $N(.)$ is the cumulative normal distribution function.

A2. The change of the value of the project on the put option

Differentiate equation (A1) with respect to the project value, V:

$$\frac{\partial P}{\partial V} = -\frac{Fe^{-r\tau}}{V\sigma_v\sqrt{\tau}}N'(d_1) - e^{-\delta\tau}N(d_2) + \frac{e^{-\delta\tau}}{\sigma_v\sqrt{\tau}}N'(d_2) \ , \tag{A4}$$

but

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-(1/2)x^2} \ , \tag{A5}$$

thus

$$\frac{\partial P}{\partial V} = -e^{-\delta\tau}N(d_2) + \frac{e^{-\delta\tau}}{\sigma_v\sqrt{2\pi\tau}}[e^{-(1/2)d_2^2} - e^{\log(F/V) + (\delta - r)\tau - (1/2)d_1^2}] \ . \tag{A6}$$

Substitute equations (A2) and (A3) in the last part of equation (A6):

$$\frac{\partial P}{\partial V} = -e^{-\delta\tau}N(d_2) + \frac{e^{-\delta\tau}}{\sigma_v\sqrt{2\pi\tau}}\left[e^{-(1/2)d_2^2} - e^{d_2\sigma_v\sqrt{\tau} + (1/2)\sigma_v^2\tau - (1/2)(d_2 + \sigma_v\sqrt{\tau})^2}\right] , \qquad \text{(A7)}$$

which simplifies into:

$$\frac{\partial P}{\partial V} = -e^{-\delta\tau}N(d_2) + \frac{e^{-\delta\tau}}{\sigma_v\sqrt{2\pi\tau}}[e^{-(1/2)d_2^2} - e^{-(1/2)d_2^2}] , \qquad \text{(A8)}$$

or:

$$\frac{\partial P}{\partial V} = -e^{-\delta\tau}N(d_2) + \frac{e^{-\delta\tau}}{\sigma_v\sqrt{2\pi\tau}}[e^{-(1/2)d_2^2} - e^{-(1/2)d_2^2}] . \qquad \text{(A9)}$$

Hence:

$$\frac{\partial P}{\partial V} = -e^{-\delta\tau}N(d_2) \le 0, \quad \text{since } \mathrm{N}(d_2) \ge 0 . \qquad \text{(A10)}$$

A3. The change of the variance of the value of the project on the put option value

Differentiate equation (A1) with respect to σ_v:

$$\frac{\partial P}{\partial \sigma_v} = Fe^{-r\tau}\left[\sqrt{\tau} - \left(\frac{\log(F/V) + \left((\delta - r) + \frac{1}{2}\sigma_v^2\right)\tau}{\sigma_v^2\sqrt{\tau}}\right)\right]N'(d_1) \qquad \text{(A11)}$$

$$- Ve^{-\delta\tau}\left[-\sqrt{\tau} - \left(\frac{\log(F/V) + \left((\delta - r) - \frac{1}{2}\sigma_v^2\right)\tau}{\sigma_v^2\sqrt{\tau}}\right)\right]N'(d_2).$$

Simplifying,

$$\frac{\partial P}{\partial \sigma_v} = \frac{Vd_1 e^{-\delta\tau}}{\sigma_v} N'(d_2) - \frac{Fd_2 e^{-r\tau}}{\sigma_v} N'(d_1) \ . \tag{A12}$$

Further,

$$\frac{\partial P}{\partial \sigma_v} = \frac{Q(d_2 + \sigma_v\sqrt{\tau})e^{-\delta\tau}}{\sigma_v} N'(d_2) - \frac{Fd_2 e^{-r\tau}}{\sigma_v} N'(d_1) \ , \tag{A13}$$

or

$$\frac{\partial P}{\partial \sigma_v} = \sqrt{\tau} V e^{-\delta\tau} N'(d_2) + \frac{Vd_2 e^{-\delta\tau}}{\sigma_v} N'(d_2) - \frac{Fd_2 e^{-r\tau}}{\sigma_v} N'(d_1) \ . \tag{A14}$$

then,

$$\frac{\partial P}{\partial \sigma_v} = \sqrt{\tau} V e^{-\delta\tau} N'(d_2) + \frac{d_2 V e^{-\delta\tau}}{\sigma_v} [e^{-(1/2)d_2^2} - e^{\log(F/V) + (\delta - r)\tau - (1/2)d_1^2}] \ . \tag{A15}$$

As we shown before, the term in the square bracket on the right-hand side of equation (A15) reduces to zero. This suggests that:

$$\frac{\partial P}{\partial \sigma_v} = \sqrt{\tau} V e^{-\delta\tau} N'(d_2) \geq 0 \ . \tag{A16}$$

A4. The change of the principal on the put option value

Differentiate the put option value with respect to *F*:

$$\frac{\partial P}{\partial F} = e^{-r\tau} N(d_1) + \frac{e^{-r\tau}}{\sigma_v\sqrt{\tau}} N'(d_1) - \frac{V e^{-\delta\tau}}{F\sigma_v\sqrt{\tau}} N'(d_2) \ , \tag{A17}$$

or:

$$\frac{\partial P}{\partial F} = e^{-r\tau}N(d_1) + \frac{e^{-r\tau}}{\sigma_v\sqrt{\tau}}\left[N'(d_1) - \frac{V}{F}e^{-(\delta - r)\tau}N'(d_2)\right] . \tag{A18}$$

Simplifying further:

$$\frac{\partial P}{\partial F} = e^{-r\tau}N(d_1) + \frac{e^{-\delta\tau}}{\sigma_v\sqrt{2\pi\tau}}\left[e^{-(1/2)d_1^2} - e^{-(d_1\sigma_v\sqrt{\tau} - (1/2)\sigma_v^2\tau) - (1/2)(d_1 - \sigma_v\sqrt{\tau})^2}\right] , \tag{A19}$$

which reduces to:

$$\frac{\partial P}{\partial F} = e^{-r\tau}N(d_1) + \frac{e^{-\delta\tau}}{\sigma_q\sqrt{2\pi\tau}}[e^{-(1/2)d_1^2} - e^{-(1/2)d_1^2}] . \tag{A20}$$

Hence,

$$\frac{\partial P}{\partial F} = e^{-r\tau}N(d_1) \geq 0 . \tag{A21}$$

A5. The change of the interest rate on the put option value

Differentiate with respect to *r*:

$$\frac{\partial P}{\partial r} = -\tau e^{-r\tau}FN(d_1) - \frac{e^{-r\tau}F}{\sigma_v\sqrt{\tau}}N'(d_1) + \frac{Ve^{-\delta\tau}}{\sigma_v\sqrt{\tau}}N'(d_2) . \tag{A22}$$

Simplifying,

$$\frac{\partial P}{\partial r} = -\tau e^{-r\tau} F N(d_1) - \frac{e^{-r\tau} F}{\sigma_v \sqrt{\tau}} \left[N'(d_1) - \frac{V}{F} e^{-(\delta - r)\tau} N'(d_2) \right] . \tag{A23}$$

From the previous section, the second term on the right-hand side is zero. Hence,

$$\frac{\partial P}{\partial r} = -\tau e^{-r\tau} F N(d_1) \le 0 . \tag{A24}$$

A6. The change of the dividend rate on the put option value

Differentiate with respect to δ:

$$\frac{\partial P}{\partial \delta} = \frac{\tau e^{-r\tau} F}{\sigma_v \sqrt{\tau}} N'(d_1) + \tau V e^{-\delta\tau} N(d_2) - \frac{\tau V e^{-\delta\tau}}{\sigma_v \sqrt{\tau}} N'(d_2) . \tag{A25}$$

This easily reduces to:

$$\frac{\partial P}{\partial \delta} = \tau V e^{-\delta\tau} N(d_2) \ge 0 . \tag{A26}$$

A7. The change of the term to maturity on the put option value

Differentiate with respect to τ:

$$\frac{\partial P}{\partial \tau} = -rFe^{-r\tau} N(d_1) + AFe^{-r\tau} N'(d_1) + V\delta e^{-\delta\tau} N(d_2) - BVe^{-\delta\tau} N'(d_2) . \tag{A27}$$

where

$$A \;=\; \frac{\left((\delta - r) + \frac{1}{2}\sigma_v^2\right)}{\sigma_v\sqrt{\tau}} - \frac{d_1}{2\tau}\;, \tag{A28}$$

and

$$B \;=\; \frac{\left((\delta - r) - \frac{1}{2}\sigma_v^2\right)}{\sigma_v\sqrt{\tau}} - \frac{(d_1 - \sigma_v\sqrt{\tau})}{2\tau}\;\;. \tag{A29}$$

Substituting *A* and *B*, equation (A29) becomes,

$$\frac{\partial P}{\partial \tau} \;=\; -rFe^{-r\tau}N(d_1) + V\delta e^{-\delta\tau}N(d_2) + \frac{\sigma_v}{2\sqrt{\tau}}Fe^{-r\tau}N'(d_1) \tag{A30}$$

$$+\frac{(\delta - r)}{\sigma_v\sqrt{\tau}}\left[N'(d_1) - \frac{V}{F}e^{-(\delta - r)\tau}N'(d_2)\right] - \frac{d_1}{2\tau}Fe^{-r\tau}\left[N'(d_1) - \frac{V}{F}e^{-(\delta - r)\tau}N'(d_2)\right].$$

Noting that the terms in the square brackets equal to zero,

$$\frac{\partial P}{\partial \tau} \;=\; -rFe^{-r\tau}N(d_1) + V\delta e^{-\delta\tau}N(d_2) + \frac{\sigma_v}{2\sqrt{\tau}}Fe^{-r\tau}N'(d_1)\;\;. \tag{A31}$$

The sign of equation (A31) is clearly indeterminate.