GROUNDWATER RECHARGE ASSESSMENT IN THE 'LANGBROEKER' AREA

Case study in The Netherlands

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1. Introduction

The main focus, worldwide, concerning the utilization of water resources is on agriculture, public water supply, industry and navigation. This attitude provides direct benefits to the stakeholders. Safeguarding of the environment and nature through optimizing the allocation of resources is often neglected.

In The Netherlands, as well, the extensive use of both the surface water and groundwaterwater in the agricultural sector, the industry and for domestic supplies have influenced nature. By using water, surface water and groundwater levels were lowered and there has been significant pollution of the water resources.

The water board by the name of ' DE Stichtse Rijnlanden' is responsible for the quantitative and qualitative surface water management of 82,000 ha of land in the southern part of the province of Utrecht.

The deterioration of the natural environment in one of the areas managed by the water board, i.e the 'Langbroeker' area, is induced by the controlled lowering of the groundwater levels in the agricultural section. Lowering of these groundwater levels in the 'Langbroeker' area supplemented by polluted water from the Kromme Rijn during dry periods has adversely influenced natural conditions.

1.1 Objectives

For surface water modeling activities of the 'Langbroeker' area, the groundwater seepage into the surface water system was an unkown parameter. Recharge and discharge areas for this area were also not identified. The study area is located at the marigine of an ice-pushed ridge, the 'Utrechtse Heuvelrug', where the groundwater levels are deep and substantial recharge may takeplace. Since the seepage in the study area and recharge from the ice-pushed ridge may be closely related, the groundwater system covering both areas has to be investigated. Therefore, the main objectives of the study are:-

I The identification of the areas with intensive seepage in the 'Langbroeker' area and its surroundings. The determination of seepage rates and their seasonal varaiations is also part of the study

II The description of the recharge regime and the assessment of principal recharge areas and rates.

In this paper the second objective is fully described. The first objective will be presented in the second paper ' Groundwater modeling for Seepage assessment'.

2.1 **Methods**

The proper establishment of recharge in the Langbroeker area and its surroundings is of vital importance for the preparation of the groundwater model and the subsequent computation of seepage rates. Groundwater recharge may be defined in a general sense as the downward flow of water reaching the water table, forming an addition to the groundwater reservoir. Recharge of groundwater may occur naturally from precipitation, rivers, canals and lakes and as a man-induced phenomenon via such activities as irrigation.

There are several methods to estimate recharge from precipitation. These can be classified as:-

1. Direct measurement over areas up to 100 m^2 .
2. Empirical methods
3. Water budget methods I) at a point; II) at a catchment's scale
4. Darcian approaches
5. Environmental or applied tracers which track the movement of particles of water in the unsaturated zone

The method of Darcian approaches that is making use of the equation of flow in the zone above the water table based on numerical models has been used to assess the recharge for the study area. The flow of water in the unsaturated zone is governed by Darcy's law.

2.2 **Equations for flow in unsaturated media**

The general equation of flow for isotropic porous media may be written as:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(k(\theta)\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(\theta)\frac{\partial H}{\partial z}\right) + \frac{\partial}{\partial z}\left(k(\theta)\frac{\partial H}{\partial z}\right) \quad 1$$

where:
θ :-moisture content
k:- hydraulic conductivity
H:-hydraulic head or soil moisture potential defined as H= z+h
z:-elevation head
h:-pressure head
t:- time

Substituting H=z+h in to the above equation yields

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left(k(\theta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(\theta)\frac{\partial h}{\partial z}\right) + \frac{\partial}{\partial z}\left(k(\theta)\frac{\partial h}{\partial z}\right) + \frac{\partial k(\theta)}{\partial z} \quad 2$$

The hydraulic conductivity varies with the moisture content or the matric pressure. The pressure head ranges from zero for saturated conditions to -10,000,000 cm for an oven dry soil. For practical purpose the decimal logarithm of the absolute value of the pressure head known as PF is often used.

Since moisture content is related to pressure head, in the soil moisture characteristic, the hydraulic conductivity relation k(θ) is also written as k(h). Equation 2 may be converted in to an equation with one dependent variable through the introduction of the differential moisture capacity:

$$\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial h}\frac{\partial h}{\partial t} = C(h)\frac{\partial h}{\partial t} \quad 3$$

Re *placing* $K(\theta)$ *by* $K(h)$ *and substituti ng equation* 3 *in to equation* 2 *yields*

$$C(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(k(h)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k(h)\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(k(h)\frac{\partial h}{\partial z}\right) + \frac{\partial k(h)}{\partial z} \quad 4$$

Equation 4 is known as Richards' equation. This equation applies for saturated as well as unsaturated flow.

A major difficulty with flow in the unsaturated zone is the nature of the relationships of hydraulic conductivity and moisture content to pressure. They are both non-linear. Many empirical relations between hydraulic conductivity and pressure have been proposed in the past in order to facilitate analytical solutions of the unsaturated flow equation. However, analytical solutions of the equation of unsaturated flow do not exist, not even for flow in only one direction. The reason is the highly non-linear character of the relation between hydraulic conductivity and pressure. Analytical solutions may only be obtained for one-dimensional flow in a homogeneous soil under special conditions.

Numerical solutions of equations for non-steady flow became possible with the advent of digital computers. Non-steady flow in porous media is almost exclusively simulated using a numerical approach. For this purpose Richards' equation could be used as it applies to transient flow in a rigid porous medium above as well as below the water table.

2.2.1 ***MUST*** *-A pseudo steady-state solution to unsaturated flow*

MUST is a model for unsaturated flow above a shallow water table. The model MUST has been selected to estimate recharge in the Langbroeker area and its surroundings since it is readily available and allows simple data management for its execution. The development of the model took place during the years 1971-1979 and a comprehensive description of the theory was presented in the form a thesis (De Laat, 1980).

The solution technique for unsaturated flow used by the simulation model MUST is based on the pseudo steady state approach. The flow situation during each time step is represented by a combination of two steady state profiles corresponding to the flux across the upper or the lower boundary of the subsoil. The time it takes before an approximate steady flow situation is reached after a change in the upper boundary flux condition depends on the rate of change, the initial soil moisture distribution, the water table depth and the soil physical properties. With shallow water tables this time varies from less than one hour for course sand to more than 10 days for loamy soils. For many flow situations an approximate steady state may be reached within one day.

The unsaturated soil profile is schematized into a root zone with a constant depth and subsoil. The flow system in the root zone is complex, largely governed by the water uptake of the roots. The root zone is considered as a reservoir, the content of which is depending on the pressure head at the interface of the root zone-subsoil. The upper boundary of the model is the soil surface and the lower boundary of the model is chosen at a fixed level below the lowest water table elevation. The upper boundary flux at the surface, q_s, is the difference between the precipitation reaching the soil surface, Ps, and the combined loss of water from crop, Et and soil Es. The structure of the model is shown in figure 1.

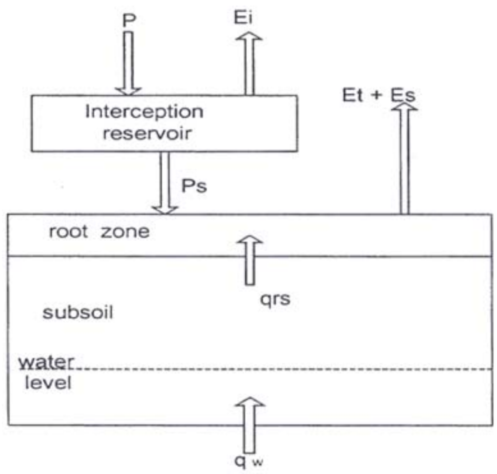


Figure 2.1 Model structure

2.3 **Input data for the model MUST**

2.3.1 *Soil physical data*

Soil physical data is the first input data for the model MUST. Soil physical data include 13 hydraulic conductivity and soil moisture values, which correspond to a standard series of 13 matric pressure values, the depth of the root zone and the thickness of each layer in root zone and subsoil if the soil is heterogeneous.

There are more than 15 different types of soil in the study area. It is time consuming to use all types of soil for the model. Therefore, grouping of soil type was necessary. The soil maps were scanned and digitized by using Map-Info professional 5.5 as a package. Soil types, which showed great similarity or covered a small area, were merged to the dominant soil type of that area. Finally four dominant soil types namely heavy clay, course sand, very fine to medium fine sand and silt on light clay have been selected. The profile of each dominant soil was taken from the same soil map.

2.3.2 *Upper boundary flux*

The soil surface is the upper boundary of the model. The net rainfall minus the actual evapotranspiration yields the flux across the soil surface. Meteorological data including precipitation, temperature, relative humidity, sunshine duration, and wind speed are necessary to determine the upper boundary flux of the model. Since the De Bilt meteorological station is within the study area, a time- series of 22 years with daily values from this station was used to compute the upper boundary flux.

2.3.3 *Time invariant data*

Other time invariant data such as date, time step, number of cases, type of boundary conditions, height of wind speed measurement, irrigation scenario, parameters to describe lower boundary conditions, initial water table, initial upper and lower boundary flux and type of land use were needed for the model.

Since for many flow situations an approximate steady state may be reached within one day, one-day time steps were used for simulation with MUST. The beginning of the calendar year was used as the starting date of simulation. The initial water table, upper and lower boundary fluxes were taken depending on the groundwater regime of the area. Since irrigation is rarely practiced, the irrigation scenario for the study area was not applied.

<u>Lower boundary conditions</u>

There are five options to describe the lower boundary condition. These are:

I) The lower boundary flux q_w is a function of the water table depth W
II) The lower boundary flux q_w is given
III) The water depth W is given
IV) The lower boundary flux is computed from given open water level data using a fixed drainage resistance
V) The lower boundary flux is computed from given open water level data using the drainage module

Option number one was selected to specify the lower boundary condition. There were no data on the water table depth and drainage resistance; therefore, options number three, four and five were omitted. Option number two, the lower boundary flux q_w, was the objective of the study so this option was also discarded.

There are three types of flux level relations available in MUST to establish the relationship between lower boundary flux and water table depth as is shown in figures a, b and c below. These relations are:

1. A hyperbolic function $$q_w = \frac{A+W}{B+C}$$

5

2. An exponential function $$q_W = A * \exp\left(B * W\right)$$

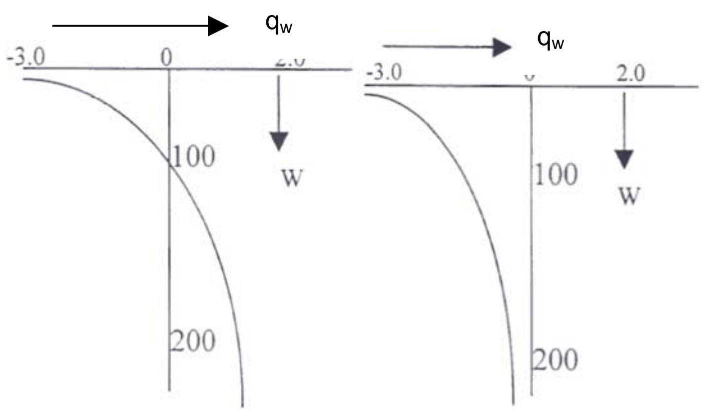
6

3. A linear function

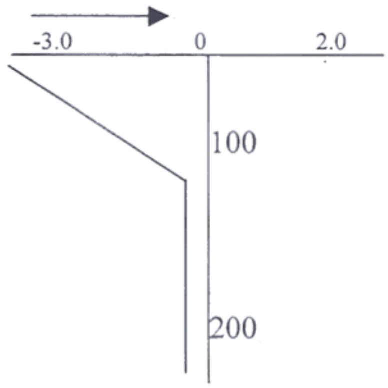
$q_W = A - \left(A/B\right) * W : if\ q_W > C : q_w = C$ 7

where: A,B,C:- empirical parameters
W:- groundwater table depth
q_w:- lower boundary flux

Three empirical parameters A, B and C are needed to establish the relation between q_w and W. The only known information to establish the relation between lower flux and groundwater level is the soil map, which shows the groundwater regime as a "grondwatertrap" of the area. The empirical parameters and type of relationship were then obtained by trial and error as discussed in section 2.4.



q_w

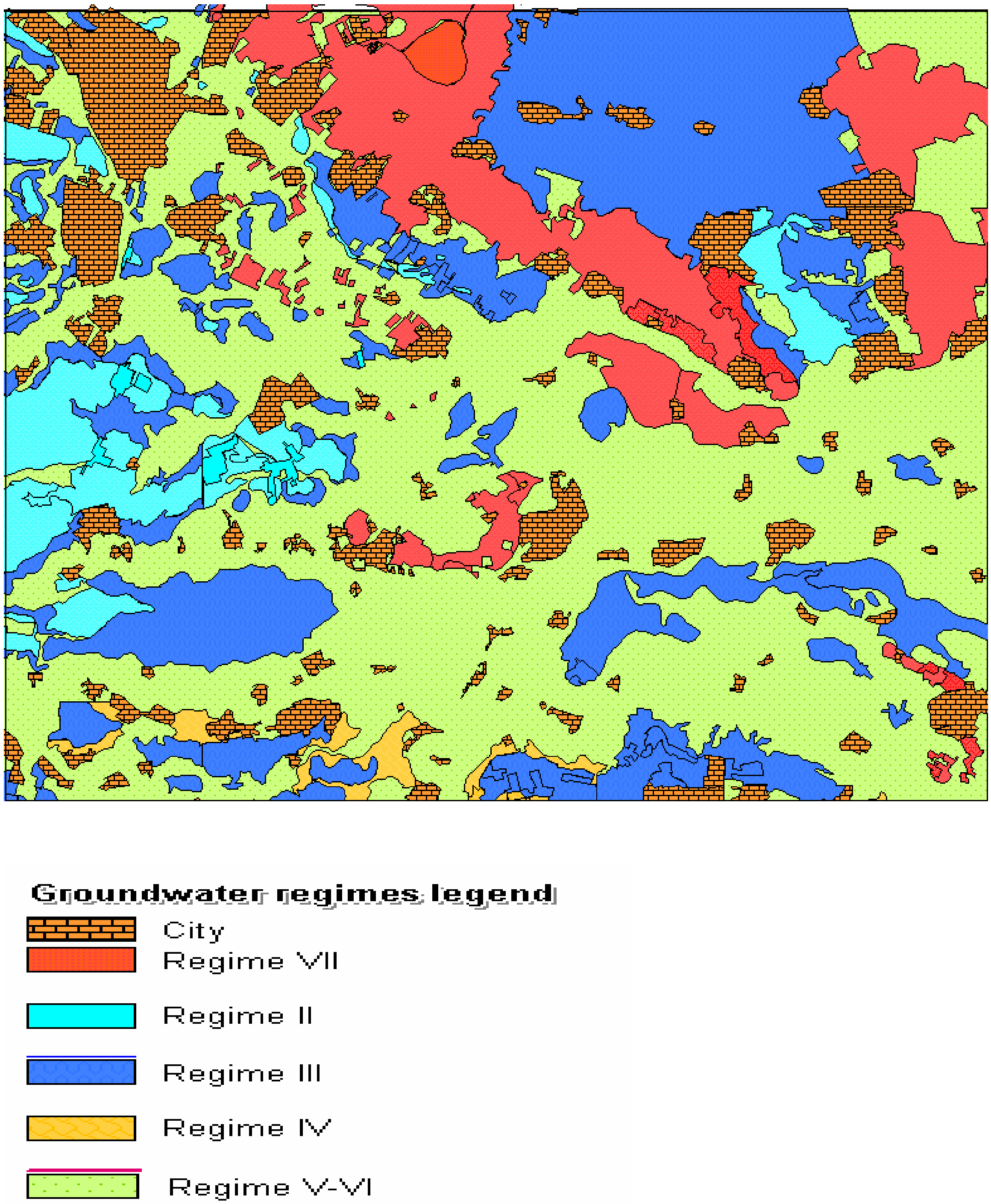


a) Hyperbolic b) Exponential c) Linear

Figure 2.4 The relationship between lower boundary flux and water table depth

<u>Groundwater regimes "grondwatertrap "</u>

In The Netherlands the shallow groundwater regimes are classified into seven "grondwatertrappen", which are described by the mean highest and the mean lowest water table. Each groundwater regime was digitized from the scanned soil map. There were only very small areas with groundwater regime I, so this regime was combined with groundwater regime II. Most areas with groundwater regime V and VI are found together so that these two regimes were combined. Groundwater regimes **II,** III, IV, V-VI and VII were the dominant groundwater regimes for the study area as shown in figure below. The larger the number of the "grondwatertrap" the deeper the water table.


Groundwater regimes legend
City
Regime VII
Regime II
Regime III
Regime IV
Regime V-VI

The three parameters A, B and C to establish the relationship between lower flux and groundwater level are depending on land use, soil type and groundwater regime. Five types of land use, four soil types and five groundwater regimes yield 100 possible combinations. However, since most combinations of land use and soil type match with a limited number of groundwater regimes, some combinations do not exist and some cover

Soil type	Land use	Ground water regime			Name
		MHW(cm)	MLW(cm)	symbol	
Heavy clay	Grass	-------	50-80	II	GrHCII
Heavy clay	Grass	<40	80-120	III	GrHCIII
Heavy clay	Grass	>40	80-120	IV	GrHCVI
Heavy clay	Grass	40-80	>120	V-VI	GrHCV-VI
Fine to medium sand	Grass	<40	80-120	III	GrFIII
Fine to medium sand	Grass	<40	80-120	IV	GrFIV
Fine to medium sand	Grass	40-80	>120	V-VI	GrFV-VI
Silt on light clay	Grass	40-80	>120	V-VI	GrLCV-VI
Heavy clay	Arable land	<40	80-120	III-IV	ArHCIII-IV
Heavy clay	Arable land	40-80	>120	V-VI	ArHCIII-IV
Fine to medium sand	Arable land	<40	80-120	III-IV	ArFIII-IV
Fine to medium sand	Arable land	40-80	>120	V-VI	ArFV-VI
Silt on light clay	Arable land	40-80	>120	V-VI	ArLCV-VI
Course sand	Mixed forest	>80	>120	VII	MXCVII
Silt on light clay	Mixed forest	<40	80-120	III	MXLCIII
Silt on light clay	Mixed forest	40-80	>120	V-VI	MXLCV-VI
-------------	Open water	---------	--------	-----	------
-------------	Urban	---------	-------	-----	-------

very small areas, so that these areas were merged to the nearest combination. Finally 19 combinations of land use, soil type and groundwater regime were obtained as shown in table 2.1

Table 2.1 19 combinations of land use, soil type and groundwater regime

2.4 **Model calibration for shallow groundwater tables**

As described in the previous sections, all input parameters except the three parameters to define the lower boundary condition were known. The three parameters A, B, and C were obtained by trial and error procedure. The groundwater regime obtained from the soil map was used as a calibration target. The model MUST was run for the first combination i.e grass on heavy clay with groundwater regime II (GrHCII) with assumed values for the three parameters A, B and C. Daily groundwater levels for 22 years were extracted from the MUST output file.

The mean highest (MHW) and mean lowest water table (MLW) were calculated for a time series of 22 years as follows. Every 14 days the groundwater table was extracted from the daily time series of the MUST output. For each year the average of the three highest and the three lowest water table depths were calculated. The average of the three highest and the three lowest water depths for each year was summed up and divided by 22, which resulted in mean highest and mean lowest water table. A small Fortran program was written to facilitate the above operations.

The calculated MLW and MHW were compared with the corresponding values on the soil map. If there was a difference between the calculated MLW and MHW with the corresponding values on the soil map, the model MUST was run for the second time with new values of A, B and C. The above procedure was repeated till the calculated value of MHW and MLW were within the range of the corresponding values of MHW and MLW from the soil map. The same procedure was followed to calibrate the lower boundary model parameters for each combination. Results of the calibrated parameter values of the lower boundary relation and the simulated MHW and MLW compared with those of the corresponding groundwater regimes are shown in table 2.2.

Table 2.2 Calibrated parameter values for the lower boundary relation and the targeted and simulated MHW and MLW values.

Name	Calibration Target		Calibrated		Calibrated Parameters		
	MHW(cm)	MLW(cm)	MHW(cm)	MLW(cm)	A	B	C
GrHCII	---	50-80	5.4	79	-0.246	54	0.28
GrHCIII	<40	80-120	35	111	-0.35	102	0.18
GrHCIV	>40	80-120	44	110	-0.46	102	0.18
GrHCV-VI	40-80	>120	50	126	-0.46	108	0.0061
GrLCV-VI	40-80	>120	46	126	-0.60	105	0.15
GrFIII	<40	80-120	35	112.8	-0.38	95	0.22
GrFIV	>40	80-120	45	112	-0.52	95	0.22
GrFCV-VI	40-80	>120	46	126.7	-0.38	105	0.006
ArLCV-VI	40-80	>120	47	133.8	-0.60	118	0.152
ArHCIII-IV	<40	80-120	41	117	-0.58	105	0.30
ArHCV-VI	40-80	>120	47	127.6	-0.53	118	0.106
ArFII-IV	<40	80-120	38.6	114	-0.53	96	0.30
ArFV-VI	40-80	>120	44	126	-0.48	108	0.093
MXL CIII	40-80	>120	35	113.7	-0.53	90	0.21
MXLCV-VI	40-80	>120	47	127	-0.56	105	0.21

2.5.2 ***Model simulation for shallow groundwater table for an average year***

Soil type	Land use	Ground water regime			Recharge (cm/d)
		MHW(cm)	MLW(cm)	symbol	
Heavy clay	Grass	-------	50-80	II	0.048
Heavy clay	Grass	<40	80-120	III	0.053
Heavy clay	Grass	>40	80-120	IV	0.058
Heavy clay	Grass	40-80	>120	V-VI	0.051
Fine to medium sand	Grass	<40	80-120	III	0.069
Fine to medium sand	Grass	<40	80-120	IV	0.076
Fine to medium sand	Grass	40-80	>120	V-VI	0.052
Silt on light clay	Grass	40-80	>120	V-VI	0.083
Heavy clay	Arable land	<40	80-120	III-IV	0.074
Heavy clay	Arable land	40-80	>120	V-VI	0.068
Fine to medium sand	Arable land	<40	80-120	III-IV	0.096
Fine to medium sand	Arable land	40-80	>120	V-VI	0.088
Silt on light clay	Arable land	40-80	>120	V-VI	0.094
Silt on light clay	Mixed forest	<40	80-120	III	0.066
Silt on light clay	Mixed forest	40-80	>120	V-VI	0.062

The model MUST was run for the average year (1990). Water table depth, upper boundary flux and lower boundary flux at the end of 1989 were taken as an initial water table depth, initial upper boundary flux and initial lower boundary flux respectively for the average year. Calibrated parameters to define the lower boundary condition were used for model simulation of the average year. The resulting recharge for each combination is shown in table 2.4.

Table 2.4 Recharge for each combination (cm/d)

2.6 **Development of a model to estimate recharge for deep groundwater tables**

The performance of the simulation model MUST is expected to deteriorate for situations with water tables at greater depth (say more than 3 m to 5 m) and for heavy soils, as the approximate steady flow situation will not be reached with in an acceptable time interval (de Laat, 1985). The model MUST was developed for the simulation of unsaturated flow above a shallow water table. However, there is no objection, from a numerical point of view, to applications with deep water tables (de Laat, 1985). Therefore, two methods have been used to estimate recharge for deep groundwater table of the study area.

These are :-

1 By using the drainage module of the model MUST
2 By using a routing method

2.6.1 ***Drainage module of the model MUST***

The lower boundary flux can be computed for given open water level data by using a fixed drainage resistance. This is the fourth option in the series of lower boundary conditions as was mentioned in section 2.3.3. Figure below shows a schematization for the drainage module when the water table is above the open water level. The lower boundary flux q_w is as follows related to the depth of the open water level below soil surface Wo(cm).

$$q_W = \frac{W - W_0}{T}$$

q_w:- lower boundary flux (recharge)
T:- drainage resistance

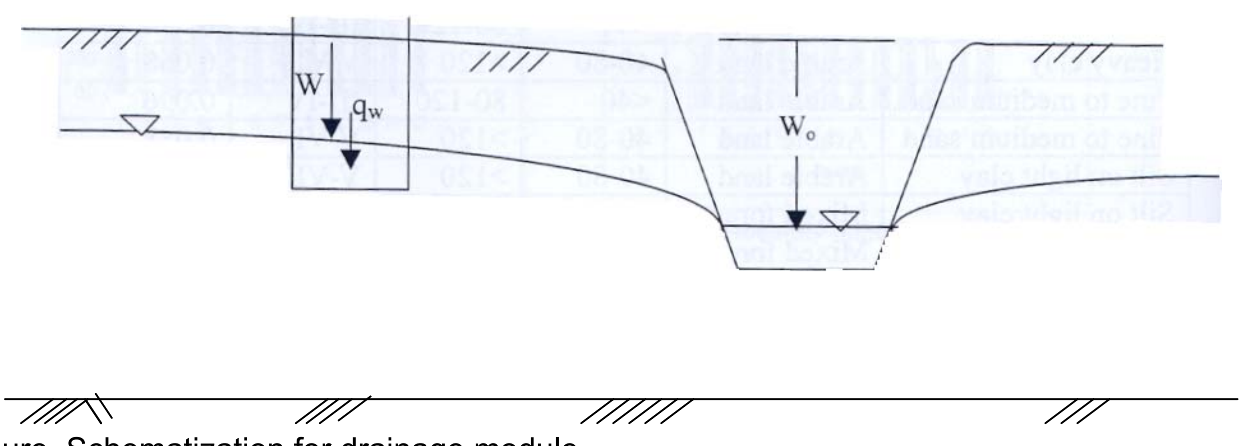


Figure Schematization for drainage module

Eight years of measured groundwater levels at two locations have been used to compare the calculated groundwater levels using the model MUST to the measured groundwater levels. The average water level of well number one is located 13 m below the surface and the other well number two is located 38 m below the surface. Both wells are located in mixed forest. The root zone depth of mixed forest is taken as 80 cm. Parameters like basic canopy resistance, capacities of the interception reservoir SIM, reflection coefficients (albedo) and the relation between day number and soil cover are taken as the average of the parameters for deciduous and coniferous forest. The parameters were used for the calculation of the evapotranspiration from forest.

The depth of the open water level below soil surface W_0 and the drainage resistance is not known for the study area. The model MUST was run for assumed value of W_0 and T. The computed groundwater levels resulting from the assumed values of W_0 and T were extracted. The computed groundwater levels were compared with the measured groundwater levels. The above steps were repeated until the difference between the calculated and measured groundwater levels were minimal. Figure 2.9 a and 2.9 b show results from the best combination of W_0 and T, which gave the minimum mean square error.

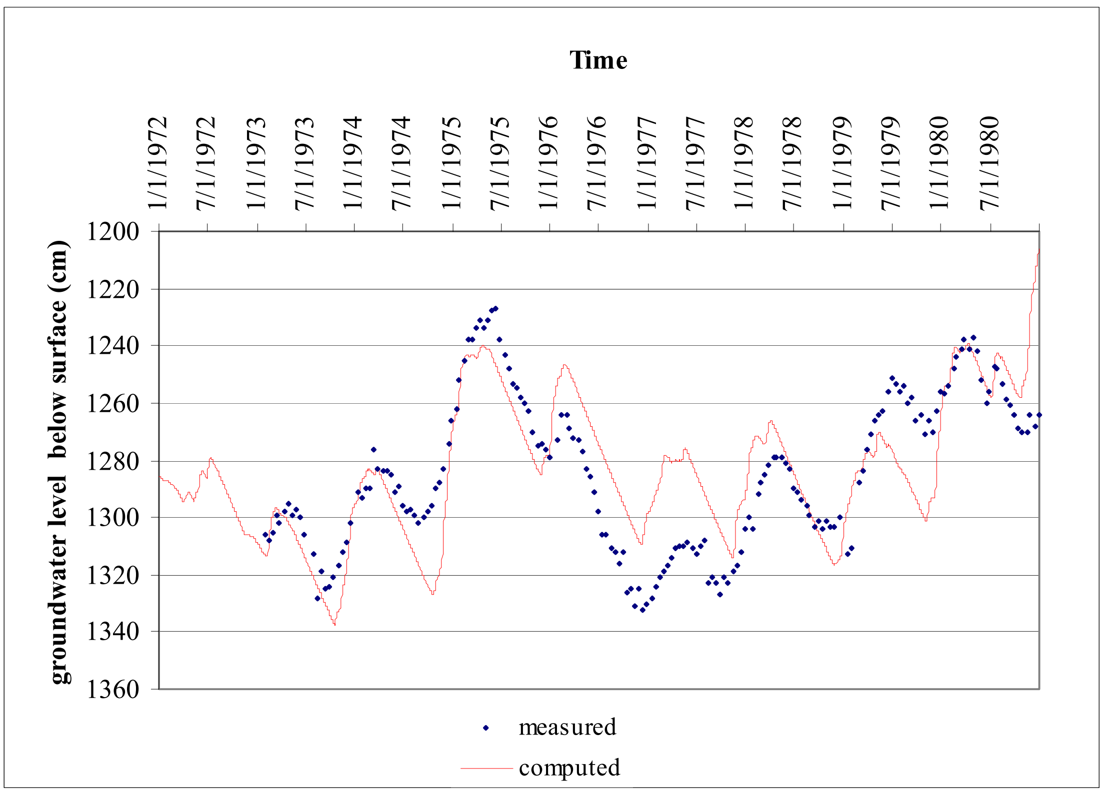


Figure 2.9a Simulation result at well number one

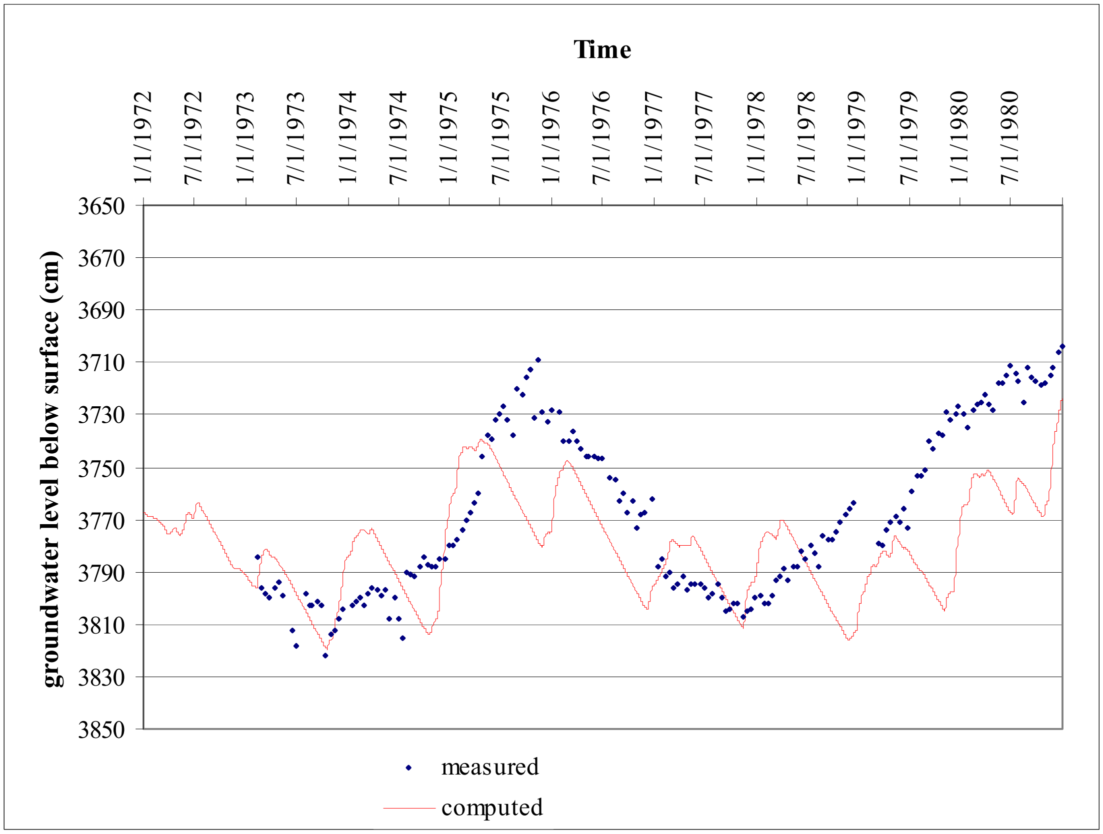


Figure 2.9b Simulation result at well number two

2.6.1 *Derivation of a routing equation*

Figure 2.10 shows a hydraulic unit (system) consisting of a reservoir that drains through a small opening. This hydraulic system converts an input signal ($q_{rs}(t)$) into an output signal $q_w(t)$. Both inputs and outputs are an observable and measurable phenomenon, which changes its magnitude in the course of time, and which has the property that it propagates. Usually the input changes its shape while passing through the system, so that one might say the system transforms the signal.

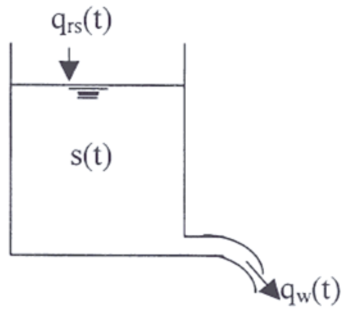


Figure 2.10 Hydraulic unit

If the storage in the reservoir is directly proportional to the output, the reservoir is said to be linear, and the proportionality constant k is known as the reservoir coefficient.
Thus

$$S\left(t\right) = k * q_W\left(t\right)$$

From the continutity equation

$$q_{rs}\left(t\right) = 0$$

For $q_{rs}\left(t\right) = 0$

$$q_W = \frac{-ds}{dt}$$

Differentiating equation 10 *with respect to t*

$$\frac{ds}{dt} = k\frac{d\,q_W}{d\,t}$$

Substituting equation 11 into equation 13; rearranging and integrating both sides gives

$$-\int_t^t \frac{1}{k}\,dt = \int_q^q \frac{1}{q}\,dq$$

The solution of the integral yields

$$q = q_0 * \exp\frac{\left(1 - t_0\right)}{k}$$

If the instantaneous input into the reservoir at time t=O equals 1, it follows from equation 10 that

$S\left(0\right) = k * q_W\left(0\right) = 1$ Or

$$q_W(0) = \frac{1}{k}$$

$Substituting\ equation\ 14b\ in\ to\ equation\ 14\ yields\ the\ impulse\ response\ of$
$a\ \sin gle\ linear\ reservoir\ U(0,t)\ which\ may\ be\ written\ as$

$$\cup(0,t) = \frac{1}{k}\exp\left(\frac{-t}{k}\right)$$

This is a function that is completely detemlined by physical characteristics of the system. This simple linear storage model was first introduced by Zoch (TNO, 1966).

The instantaneous unit hydro graph (impulse response) of the single linear storage can be convoluted with a constant input rate qrs(1).

$$q_W(t) = q_{rs}(1)\int_0^1 \frac{1}{k}\exp\left(-\left(\frac{t-t}{k}\right)\right)dt = q_{rs}(1)\exp\left(\frac{t-t}{k}\right)\int_t^t$$

$$q_w(t) = q_{rs}(t)\left\{1-\exp\left(\frac{-t}{k}\right)\right\}$$

At the end of the first time interval of unit duration t= 1

$$q_w(1) = q_{rs}(1)\left\{1-\exp\left(\frac{-t}{k}\right)\right\}$$

Since the principle of superposition is valid for the impulse response, the output flow rate at the end of the next unit interval of inflow rate is

$$q_w(2) = q_w(1)\exp\left(\frac{-1}{k}\right) + q_{rs}(2)(1-\exp\left(\frac{-1}{k}\right))$$

Thus for a simple linear storage characterised by its proportionality factor k, the outflow rate at the end of an interval can be derived from the outflow rate at the end of the former interval and the inflow during the considered interval. In general:-

$$q_w(t) = q_W(t-1) * \exp\left(\frac{-1}{k}\right) + q_{rs}(t) * \left(1-\exp\left(\frac{-1}{k}\right)\right)$$

This equation is known as a routing equation.

2.6.2 *Routing the recharge in the presence of a deep water table*

MUST solve the (downward) flow in the subsoil as a succession of steady states. It is obvious that for deep water tables a steady state will not be reached in a time span of a simulated with the linear routing method, which was derived in the previous section, and now written as:

$$q_{w}(t) = q_{W}(t-1) * \exp\left(\frac{-1}{k}\right) + q_{rs}(t) * \left(1 - \exp\left(\frac{-1}{k}\right)\right)$$

Where
$q_w(t)$:- recharge at time t
$q_w(t-1)$:- recharge at t-1
K:- proportionality factor
q_{rs}:- out flow from the root zone at time t

The groundwater level changes with the recharge from the unsaturated zone and the discharge from the saturated region in to the surface water system as shown in Fig below. Neglecting the recharge, the relation between the piezometric level h and the drainage of groundwater into the surface water system q_b depends on the open water level d, the resistance to flow in the aquifer α and the drainable porosity ε.

The following linear relation between the drainage to the surface water system q_b (contributing to the base flow of the river) and the piezometric level h is assumed.

$$h - d = \frac{a}{\varepsilon} q_b$$

Where

h:- Piezometric surface above the reference level,
d:- Open water surface above the reference level,
q_b:- Drainage from the aquifer into the surface water system as a discharge per surface area
α:- Characteristic of the saturated media and
ε:- Drainable porosity

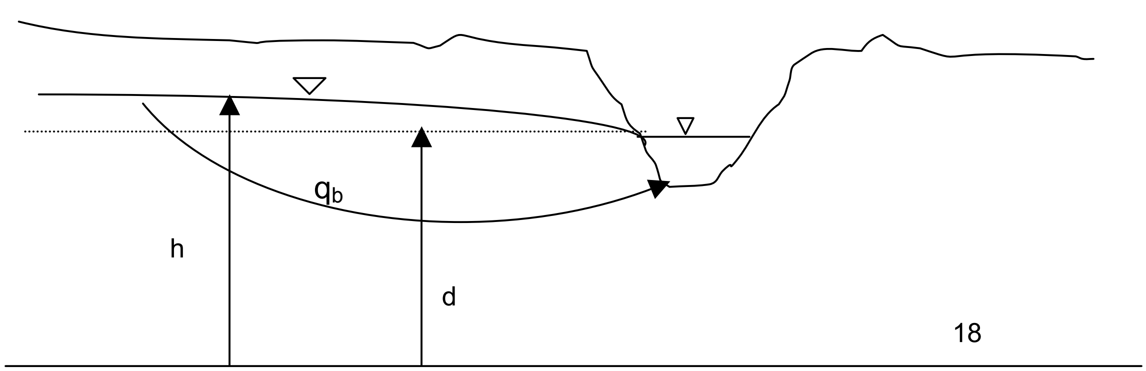


Figure Typical drainage situation

For a vertical soil column shown in Fig above the continuity equation may be written as:

$$q_b = -\frac{ds}{dt} = -\varepsilon \frac{dh}{dt}$$

where the drainable porosity ε is assumed to be constant in time.

Combining equation 22 and 23 yields:-

$$-\frac{dh}{dt} = \frac{h-d}{a}$$

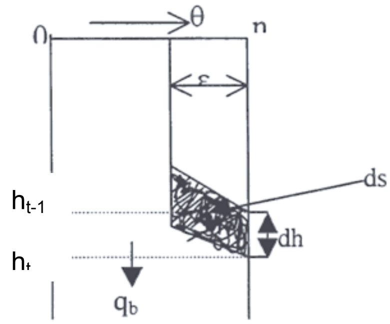


Figure 2.12 Vertical soil column

Rearranging and integrating both sides of equation 24 yields

$$\int_{h_0}^{h} \frac{1}{h-d}\, dh = \int_{t_0}^{1} \frac{1}{a}\, dt$$

$$h_t - d = (h_0 - d) * \ell \frac{-(1-t_0)}{a}$$

For an instantaneous unit rise of the water table (h_o - d= 1) at time t_o=0 the impulse response equation for the drawdown may be written as

$$H(0,t) - d = e^{\frac{-t}{\alpha}}$$

The recharge of the water table q_w is related to the water table rise as follows

$$\frac{dh}{dt} = \frac{q_w}{\varepsilon}$$

Convoluting equation 25b. with a continuous rise of the water table gives

$$h_t - d = \frac{dh}{dt}\int_0^t e^{\frac{-(t-\tau)}{\alpha}} d\tau$$

Substituting equation 26 into equation 27 and solving the integral gives

$$h_t - d = \frac{q_w}{\varepsilon} * \alpha\left(1 - e^{\frac{-t}{\alpha}}\right)$$

Routing equation 25a for the drawdown of the water table due to discharge as

$$h_t - d = (h_{t-1} - d) * e^{\frac{-t}{\alpha}}$$

Combining equation 29 with the effect of recharge on the groundwater level as given by equation 28, yields the following expression for the computation of the piezometric level.

$$h_t = (h_{t-1} - d) * e\,\frac{-t}{a} + \frac{a\,q_w}{\varepsilon}\left[1 - e\,\frac{-t}{a}\right] + d$$

2.6.3 *Application of the routing model*

Eight years of measured groundwater levels at two locations have been used to verify the routing method. The model MUST was run to obtain the outflow from the root zone, which subsequently was routed to the groundwater reservoir. For the lower boundary condition option number III, groundwater level is given, was selected. Both sites where the groundwater level was measured are located in a mixed forest.

The outflows from the root zone simulated with MUST have been routed to the groundwater reservoir by using equation 2.20. Equation 2.30 has been used to calculate the groundwater level at any time t. The aquifer drainage resistance (α), depth of the aquifer (d) and the routing parameter (k) have been optimized by using the optimization algorithm Solver in Excel. The value of the effective porosity, which is 0.16, was kept constant throughout the optimization. Many trials were carried out by changing the parameters α, k, and d to fit the computed and measured groundwater levels. For a relatively shallow groundwater table (well one) k=58 days, α=205 days and d=1344 cm were found to be the best, based on the minimum mean square error between the observed (two observation per month) and simulated levels. For deep groundwater tables (well two) k=198 days, α=587 days and d=3920 cm gives minimum mean square error. Figure 2.13 shows the results of computation using the above values of k, α and d.

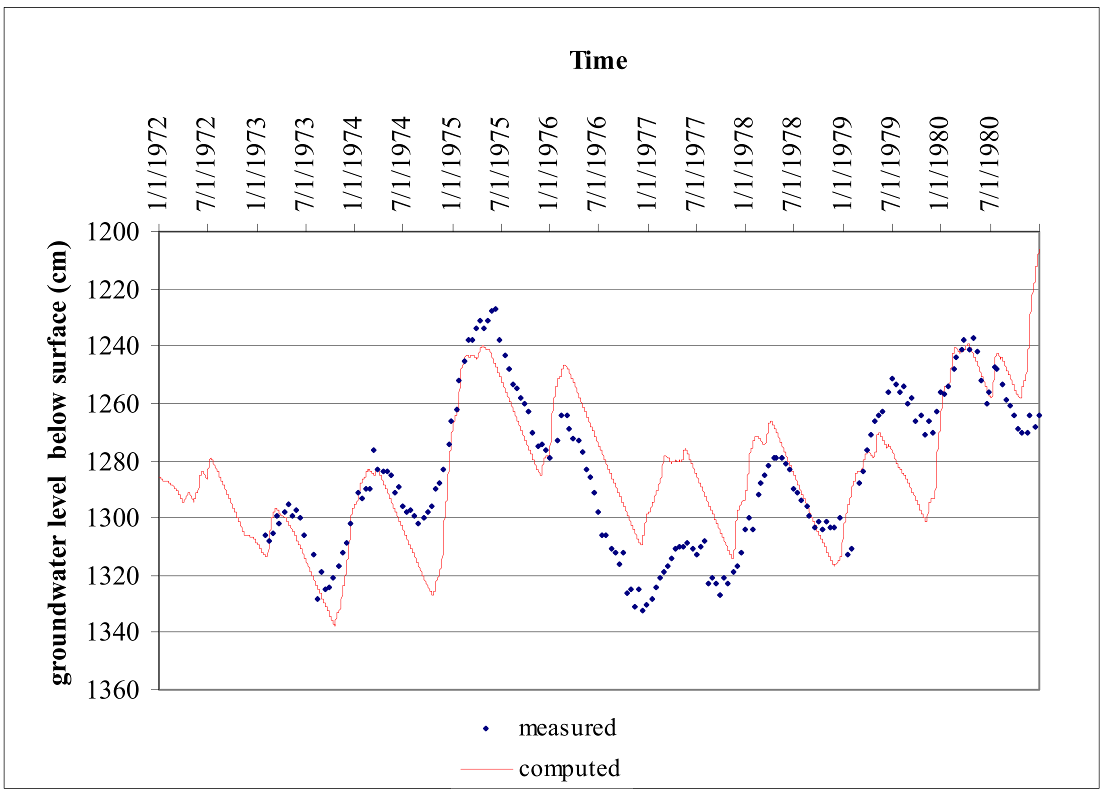


Figure 2.13a) Computed and measured groundwater level at site one

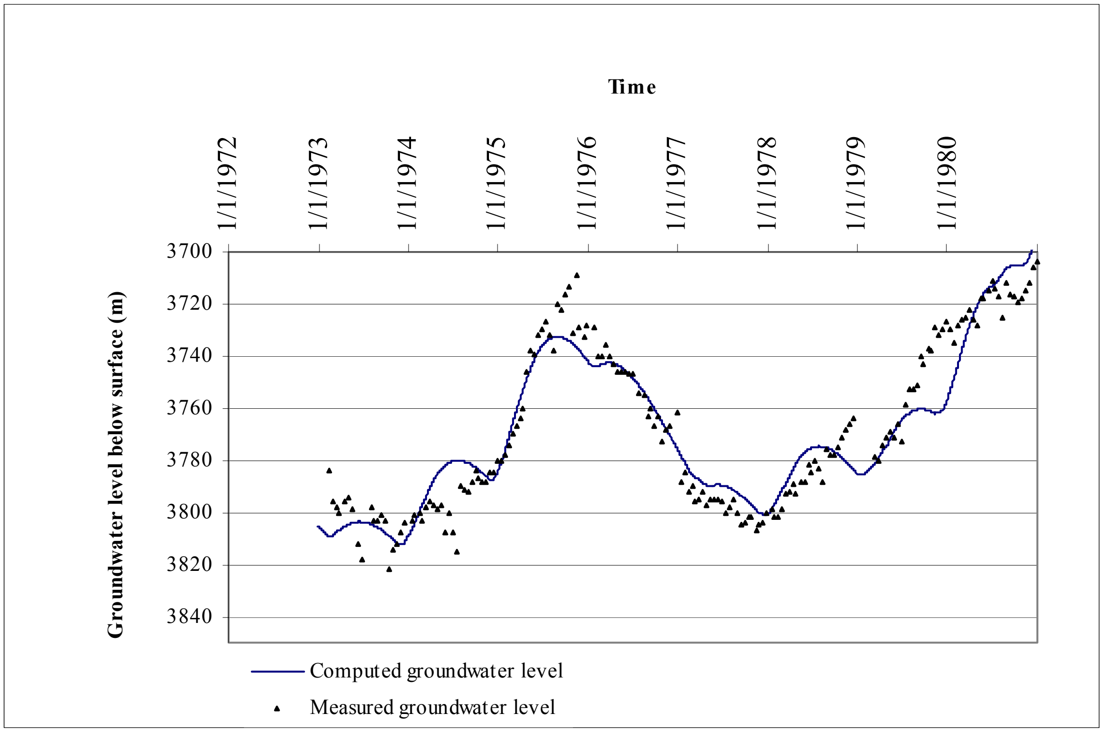
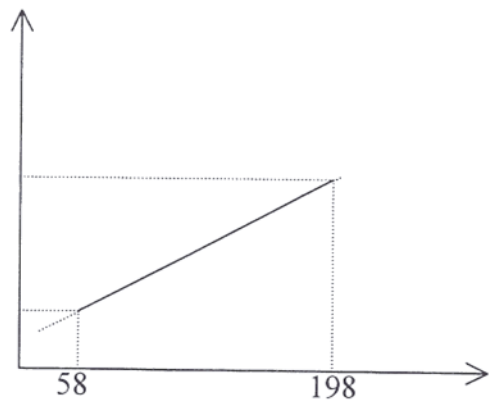


Figure 2.13b) Computed and measured groundwater level at site two

A comparison has been made between the two methods. As it was shown in figure 2.9a and b and figure 2.13a and b, the routing method gives better fit for both wells. Therefore, the routing method has been selected to assess the recharge in the presence of deep groundwater levels in the study area.

2.6.4 *Application of the routing method for an average year (1990)*

Attempts were made to establish a relationship between the routing parameters, the aquifer resistance and depth to the drainage base. However, due to lack of data and time it was not possible to establish this relation. It is clear from the two optimized values of k that as groundwater level from the surface increases the proportionality factor also increases but is not known how these parameters are related. Proportionality factor was assumed to vary linearly with depth to the groundwater level as shown in figure 2.14.



Proportionality factor 4

Figure 2.14 Relation between proportionality factor and depth

Measured groundwater levels for the year 1990 were interpolated by using the Kriging method and Surfer as a software package. A class interval of 5 m was used. For each class interval the proportionality factor was obtained from the above chart. The outflow from the root zone was extracted from the model MUST and routed to yield the recharge. Table 2.5 shows the calculated recharge for each class interval.

Table 2.5 Recharge for each class interval for mixed forest

Depth below surface (m)	5-10	10-15	15-20	20-25	25-30	30-35
Recharge (cm/d)	0.017	0.024	0.030	0.036	0.041	0.045
Depth below surface (m)	35-40	40-45	45-50	50-55	55-60	
Recharge (cm/d)	0.047	0.049	0.051	0.052	0.053	

3 Conclusions and Recommendations

With regard to the second objective

- The model MUST was capable to perform the unsaturated flow modeling to assess the recharge in the study area where the groundwater levels are shallow.
- In the routing method, the relationships between different parameters in the routing equation is not well known. Therefore, further investigation is needed to assess the recharge in the study area where the groundwater levels are deep.