Financial Frictions and Exchange Rate Regimes in the Prospective Monetary Union of the ECOWAS Countries

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Abstract: This paper aims to explore the connection between external borrowing constraints and exchange rate regimes in the prospective currency union of the Economic Community of the West African States (ECOWAS). We use a small open economy Dynamic Stochastic General Equilibrium (DSGE) model embodying several frictions to capture pervasive features pertaining to developing countries, including operating costs in firm’s capital utilization, imperfect capital mobility due to borrowing constraints, inefficiencies in investment and absorptive capacity constraints. The financial accelerator mechanism à la Bernanke et al (1999) is explicitly modeled where domestic firms’ external borrowing premium is a function of the state of their balance sheets. We employ Bayesian estimation methods on annual data to partially estimate the model parameters for each of the ECOWAS countries. The model is then used to assess the role of external borrowing constraints and of the other frictions in choosing a monetary policy regime for the future Central Bank of the ECOWAS. We find that stabilization-wise, the insulating role of flexible rates in the presence of adverse foreign shocks is dominant than that of fixed rates for each country.

Keywords: External Borrowing Constraints, Monetary Policy Regime, ECOWAS

JEL Classification: E3, E4, F3

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1. Introduction

The prospective monetary arrangements between 13 countries of the Economic Community of the West African States (ECOWAS)\(^2\) have taken center stage in the debate. The plan towards the creation of the currency union is in a transitory phase since 2003 with the creation of the West African Monetary Zone (WAMZ),\(^3\) which later is meant to merge with the West African Economic and Monetary Union (WAEMU).\(^4\) 2005 was initially due to be the commencement date of the monetary unification but the process was postponed for 2009 and has never been effective since. At the core of this delay is the sake for increased degree of preparedness by allowing member States meet *inter alia* convergence criteria around inflation, official reserves and public deficit. A West African Monetary Institute was created in 2000 in order to monitor compliances with the convergence criteria and organize macroeconomic surveillance within the group. Therefore, the poor achievement of the convergence criteria turns out to be one of the stumbling blocks impeding the progress towards the creation of the single currency by the ECOWAS.

Whether the future currency union of the ECOWAS countries would be beneficial or detrimental for the member countries has been debated from many points of view in a large body of literature summarized in Diop et Fall (2011). These studies, though not in DSGE framework, investigate the rational and the feasibility of single currency with respect to the following conditions: the symmetry or similarity of shocks across countries (Debrun et al 2005; Bénassy-Quéré et Coupet, 2005; Tsangarides et Sureshi, 2008; Xiaodan et Yoonbai, 2009; Dufrenot, 2009a); trade and financial integration (Gbetnkom, 2006; Goretti et Weisfeld, 2008 ; Masson, 2008 ; Sy, 2008); coordination of the macroeconomic policies with regards to individualist and free rider behavior across countries (Debrun et al., 2005 ; Masson et Patillo, 2001, 2002); and convergence criteria around nominal aggregates (Alagidede et al., 2008, Dufrenot, 2009b).

When exogenous shocks hitting member countries are similar and countries are more open and well-diversified in their economies, this offers stance for increased degree of factor mobility among countries and the flexibility of their wages and prices, ultimately lowering the cost from participating in a currency area (Xiaodan and Yoonbai (2009)). Bangaké (2008) investigates empirically the relation between bilateral

\(^2\) ECOWAS encompasses 15 members, namely Benin, Burkina Faso, Cape-Vert, Cote-d’Ivoire, The Gambia, Ghana, Guinea, Guinea-Bissau, Liberia, Mali, Niger, Nigeria and Sierra Leone, Senegal and Togo. Of the 15 founding members of the ECOWAS, two countries including Liberia and Cape-Verde declined to participate in the currency union.

\(^3\) The WAMZ comprises five countries including The Gambia, Ghana, Guinea, Nigeria and Sierra Leone.

\(^4\) The WAEMU is a subset of the ECOWAS and encompasses 8 countries, namely Benin, Burkina Faso, Cote-d’Ivoire, Guinea, Mali, Niger, Senegal and Togo.
exchange rate volatility with regard to the criteria of the theory of optimum currency area in a sample of Sub-Saharan African countries and suggests that African countries are to set up a currency union but advocates for a progressive enlargement from the existing regional economic groupings. Dupasquier et al. (2005) confirm the same finding in a rigorous theoretical framework. A sample empirical study has emphasized the role of a single currency union in the ECOWAS as a catalyst for deepening trade and financial integration (Gbetnkom, 2006; Goretti and Weisfeld, 2008; Masson, 2008; Sy, 2008).

Developing economies often finance accumulation of physical capital by issuing foreign currency denominated debt (original sin). The World Bank Debtor Reporting System (DRS) confirms that in 2010 around 30 percent of developing countries’ external net debt inflows are denominated in developed countries currencies such as the U.S dollar. When credit-constrained firms’ assets are denominated in domestic currency while liabilities are denominated in foreign currency, an exchange rate depreciation, which is likely to occur in the case of pegged exchange regime, wreaks havoc with their net worth by raising the debt burdens and making it more expensive to repay. As a result of that, firms’ costs of capital increase, leading to a contraction in equilibrium investment. This effect of foreign currency debt on corporate balance sheets and the ensuing contractionary effects have been empirically discussed in Calvo (2002) and Calvo and Reinhart (2000). Conversely, some empirical studies in the framework of sticky price model provide evidence that currency devaluation could be expansionary in the short-run due to its demand-switching effects (see Cespede et al. 2004 among others). Therefore, the compounding effect from collapse in investment through “balance sheet effect” and improved current account balance ultimately translates into effects on the economy.

In this study we attempt to formally assess the above claims by introducing the financial accelerator a la Bernanke et al (1999) into an otherwise standard small Open economy New Keynesian DSGE model. In the model, credit-constrained firms are exposed to foreign currency denominated debt and their borrowing constraints depend on the state of their balance sheets such that, a currency depreciation is likely to increase the firms’ insolvencies and costs of capital as well as to deter optimal investment. We partially calibrate and estimate the model with data in order to mimic as close as possible the structural features of the five founding members of the WAMZ as well as the WAEMU region. Specifically, we compare the

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5 Expenditure switching effects affect external balance in that it is the substitution between export and import demand. When domestic price of imports increases while foreign price of exports decreases, this decreases imports and increases exports. Therefore, devaluation leads to an improvement in current account balance respective of the elasticity of substitution between imports and exports.
different exchange rate regimes of each country under a shock of “external finance premium” (the difference between the cost of funds raised externally and the opportunity costs of funds internal to the firm) to pin down the responses of key variables of interest in terms of their cyclical properties, using impulse response functions.

Studies that have linked financial distress with exchange policy are somewhat scant. Recent studies carried out by Cook (2004), Eleckdag and Tchakarov (2007), Cespedes et al. (2004), Devereux et al. (2006) and Gertler et al. (2007) incorporate these frictions to analyze different monetary policies. The results of these studies allow us to classify them into two groups: Cook (2004), Eleckdag and Tchakarov (2007) found a greater role for the fixed exchange rate in macroeconomic stabilization of the emerging economies, while Cespedes et al. (2004), de Devereux et al. (2006) and Gertler et al. (2007) emphasize the primacy of the flexible exchange rate regime on the fixed exchange rate, which is consistent with the recommendation of the standard Mundell-Fleming framework. From the theoretical point of view, these studies are subject to criticisms because they assume a complete exchange rate pass through, perfect mobility of capital and flexible domestic import prices. Empirical evidences by Akofio-Sowah (2009) on SSA developing countries and on Latin American countries point to incomplete pass-through as a result of low inflation environment. Within the SSA region, the Common Market for Eastern and Southern Africa (COMESA) countries have the highest inflation and therefore the exchange rate pass-through in those countries is 25 to 50 percent higher than that in the WAMZ, the WAEMU and the Central African Economic and Monetary Community (CEMAC). In the same vein, Diop et Fall (2011) also assume incomplete pass-through in a dynamic stochastic general equilibrium model to study which exchange rate regime would be relevant for the ECOWAS members. They find that fixed regime is likely to foster more business cycle stability without undermining growth performances of those countries.

In line with Akofio-Sowah (2009), this study assumes an incomplete pass-through through the introduction of staggering price adjustments in the model. Furthermore, apart from Gertler et al. (2007) and Sangaré (2013), none of the previous studies has taken into account both risk premium on the balance sheet of firms and country risk premium in the modeling of the financial sector. Empirically, the emerging economies have been the ones that have attracted the most growing research interest on exchange rate policy following the series of crises that hit those countries in the 1990s and the 2000s (Thailand, Indonesia and
South Korea, 1997; Russia and Brazil, 1998; Argentina and Turkey, 2000; Turkey, 2001; and Argentina, 2002 among others).  

To the best of our knowledge, there has been very limited empirical application of such framework for exchange rate policy, let alone the connection between credit market frictions and exchange rate regimes in SSA regions. Except for the study of Gertler et al. (2007) on emerging countries Asian countries, no study addressed the relationship between constraints to domestic firms’ borrowing in international credit market and exchange rate regime in the context of Africa. The rational for applying this framework in the context of typical developing countries such as those of SSA is twofold. First, these countries are credit-constrained and capital-scarce. Second, they cannot use their own currency when borrowing in international capital market, which is also known as the “original sin”.

The rest of the paper is structured as follows. Section 2 describes the extended new Keynesian open economy dynamic stochastic general equilibrium model with financial frictions. Section 3 explains the calibration and econometric strategies used to estimate the parameters of the model, the estimation results and impulse response functions of the shocks. Finally, section 4 concludes.

2. The Model

The core framework in this paper is a typical new Keynesian small open economy DSGE model with nominal rigidities which is the key for monetary policy. It builds on Sangaré (2013) with some similarities on these aspects: First it accounts for incomplete pass-through (Monacelli, 2005). Second, it includes the financial accelerator mechanism a la Bernanke et al. (1999)—hereafter BGG—by linking domestic firms’ borrowing conditions—the cost of capital induced by the risk premium—to the state of their balance sheets. The original sin comes about to the extent that the framework considers that an important part of the debt is denominated in foreign currency. Through the borrower balance sheets, the financial accelerator mechanism works to amplify and ensure the persistence of shocks to the economy. Third, the model assumes imperfect capital mobility. We then extend on these model features to include habit formation in consumption utility (Justiniano and Preston, 2004) to allow for smoothed consumption path and to avoid for unrealistically drastic adjustments (Christiano et al. 2005). Furthermore, the extended model exhibits two types of firms encompassing firms adopting forward-looking behavior on the one hand, and firms endowed

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6 Eichengreen and Hausmann, 1999; Cook, 2004; Devereux et al. (2006) and Gertler et al. (2007) are inter alia a sample studies on these countries.
with backward-looking behavior in price setting on the other. The previous study considers only the presence of forward-looking firms while overlooking the backward-looking behavior of some firms. In addition, while the previous study focuses on emerging Asian countries, this paper is interested in linking financial frictions with the choice of monetary policy regime in the prospective currency union of the founding members of the ECOWAS or a subset of countries of this.

The framework contains the salient features of the standard new Keynesian small economy DSGE model with respect to the optimizing behavior of the microeconomic units, entrepreneurs, capital producers and household, government, the monetary authority and a foreign sector. Households supply labor to entrepreneurs and consume tradable goods that are produced both domestically (H) and abroad (M). Credit-constrained firms borrow in foreign currency and in domestic currency (see chart 1). Their demand for capital depends on their net worth via payment of a risk premium. This is the key aspect of the financial accelerator.

Chart 1: Flow Chart of the Economy

A continuum of monopolistically competitive firms (retailers) operating through domestic and foreign market set their prices in the local market on a staggered basis a la Calvo (1983). This helps to explain
inflation inertia and output persistence. Capital accumulation is subject to adjustment costs. The law of one price is introduced in the model to account for the assumption of incomplete pass-through which further adds wrinkles to the analysis.

2.1. Households

The domestic small open economy is populated by a continuum of infinitely-lived maximizing households. The intertemporal utility function of the households depends positively on consumption $C_t$ relative to an external habit formation $hC_{t-1}$ and negatively on labor supply $L_t$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{(L_t)^{1+\eta}}{1+\eta} \right)$$  \hspace{1cm} (1)

Where $0 < \beta < 1$ is the discount factor; $\sigma > 0$ is the coefficient of relative risk aversion (or inverse of the intertemporal elasticity of consumption, and $\eta > 0$ is the inverse elasticity of labor supply; $C_t$ is a CES function defined over domestic goods and imported goods:

$$C_t = \left[ (1 - a)^{\frac{1}{\theta}} (C_{H,t})^{\frac{\theta-1}{\theta}} + a^{\frac{1}{\theta}} (C_{M,t})^{\frac{\theta-1}{\theta}} \right]^\frac{1}{\theta-1}$$  \hspace{1cm} (2)

Where $C_{H,t}$ and $C_{M,t}$ stand for the usual CES aggregators of the quantities of domestic and foreign goods respectively, and $\theta > 0$ is the elasticity of substitution between both types of goods; $0 < a < 1$ is the share of foreign-produced goods in the consumption bundle:

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j) \frac{x}{x} \, dj \right)^{\frac{x}{x-1}} \text{ and } C_{M,t} = \left( \int_0^1 C_{M,t}(j) \frac{x}{x} \, dj \right)^{\frac{x}{x-1}} ,$$

$\chi > 1$ is the elasticity of substitution between the different varieties of goods and $C_{H,t}(j)$ stands for the consumption of the variety $j$ of the domestic and foreign good. The consumer price index associated with equation (2) is defined as:

$$P_t = \left[ (1 - a)\left( P_{H,t} \right)^{\theta-1} + a\left( P_{M,t} \right)^{\theta-1} \right]^{\frac{1}{\theta-1}}$$  \hspace{1cm} (3)

In the same vein, the corresponding aggregate prices over the varieties $j$ of domestic and foreign goods are given by:
\[ P_{H,t} = \left( \int_0^1 P_{H,t}(j)^{X-1} \, dj \right)^{\frac{1}{X-1}} \text{ and } P_{M,t} = \left( \int_0^1 P_{M,t}(j)^{X-1} \, dj \right)^{\frac{1}{X-1}}. \]

Optimal allocation of expenditures between domestic and foreign goods can be written as

\[
\min_{c_{H,t}, c_{M,t}, v_t} P_{H,t}c_{H,t} + P_{M,t}c_{M,t} = P_t c_t
\]

s.t. \[ c_t = \left( 1 - \alpha \right) \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} c_t + \alpha \left( \frac{P_{M,t}}{P_t} \right)^{-\theta} c_t \]

This expenditure minimization on domestic and foreign goods yields the demand functions for domestically produced and imported goods as in the following:

\[
C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} c_t; \quad C_{M,t} = \alpha \left( \frac{P_{M,t}}{P_t} \right)^{-\theta} c_t
\] (4)

The household budget constraint is given by

\[ P_t c_t + R_{t-1}B_{t-1} + R_{t-1}^w \Psi_{D,t-1} S_t D_{H,t-1} + \tau_t = W_t L_t + B_t + S_tD_{H,t} + \Lambda_t + T_t \] (5)

Following Devereux et al. (2006), we assume that households purchase public bond in local currency \( B_t \) at a nominal interest rate \( r_t = R_t - 1 \), and that part of their debt is denominated in foreign currency, \( D_{H,t} \).

The nominal interest rate associated to the latter debt is \( r_t^w(\Psi_{D,t}) = (R_t^w - 1)(\Psi_{D,t}) \), where \( \Psi_{D,t} \) stand for the country borrowing premium (detail description of that follows later in this section). We introduce this country borrowing premium to account for the assumption of imperfect international capital mobility and partly for technical reasons on the stationarity of the total net foreign indebtedness (Schmitt-Grohe and Uribe 2001). Following Sangaré (2013), the country borrowing premium is a modified version of Adolfson et al. (2008) as follows:

\[ \Psi_{D,t}(d_t, Z_t) = \exp \left( \psi_D \left( \frac{S_t D_t}{Y_P t} + Z_t \right) \right), \]

Where \( d_t = \frac{S_t D_t}{Y_P t} \) is the total debt to GDP ratio in period \( t \). \( \Psi_{D,t} \) is an increasing function of the total net foreign indebtedness \( \left( \Psi_{D,t} \right)_d > 0 \) and \( \Psi_{D,t}(0,0) = 1; D_t \) is total debt of the country and comprises \( D_{H,t} \) (the households foreign debt) and \( D_{E,t} \) (the entrepreneurs foreign debt) \( (D_t = D_{H,t} + D_{E,t}) \). We
elaborate on $D_{E,t}$ in the next sections; $\psi_D$ is the elasticity of the country’s borrowing premium with respect to the debt and $Z_t$ stands for a random shock:

$$Z_t \sim AR(1), \quad \log(Z_t) = \zeta_z Z_{t-1} + \varepsilon_{z,t}, \text{ with } \varepsilon_{z,t} \sim i.i.d(o, \sigma^2).$$

Besides the financial borrowing, the flow of the households’ income is composed of nominal wages $W_t$ from labor services and profits $\Lambda_t$ of monopolistically competitive firms they own. They also receive transfers $T_t$ from government, which represents the lump sum tax payment $\tau_t$. $S_t$ stands for the nominal effective exchange rate and $R_{t-1} B_{t-1} + R_{t-1}^w \Psi_{D,t-1} S_t D_{H,t-1}$ is the total gross refund on the borrowings contracted by the households at $t-1$.

The representative household chooses the set $\{C_t, L_t, B_t, D_{H,t}\}^\infty_0$ that maximizes its intertemporal utility (1) subject to its budget constraint (5). The first order conditions of the maximization problem are given by:

$$\frac{(L_t) \eta}{Q_t} = \frac{W_t}{P_t}$$

$$1 = \beta R_t E_t \left( \frac{Q_{t+1}}{P_{t+1}} \right)$$

$$Q_t = \beta R_t^w \Psi_{D,t}(d_t, Z_t) E_t \left( \frac{Q_{t+1}}{P_{t+1}} \frac{S_{t+1}}{S_t} \right)$$

Where $Q_t = (C_t - hC_{t-1})^{-\sigma}$

The first order conditions of the consumer’s problem are standard and can be written in a log-linearized form as:

$$w_t - p_t = \eta L_t + \frac{\sigma}{1 - h} (C_t - hC_{t-1})$$

$$C_t = \frac{h}{1 + h} C_{t-1} + \frac{1}{1 + h} E_t C_{t+1} - \frac{1 - h}{\sigma (1 + h)} (r_t - \pi_{t+1})$$

Where $\pi_{t+1}$ is the next period’s overall inflation in the economy defined as $P_{t+1} - P_t$. Condition (9) and (10) can be viewed as the marginal rate of substitution between consumption and labor while (8) is the famous Euler equation of consumption. Combining equations (7) and (8) yields the usual condition of the Uncovered Interest Parity (UIP) adjusted for the risk premium.
2.2. The real exchange rate, the terms of trade, and incomplete pass-through

One of the recent developments in open economy New Keynesian DSGE is the modeling of the deviation of prices from the Law of one price referred to as the law of one price gap (Monacelli, 2005). The claim is that monopolistically competitive firms exert some power on price of goods they import and distribute creating a distortion between the domestic and foreign prices of these imported goods when expressed in the same currency. It is this distortion that is referred to as the law of one price gap. It is assumed that the Law of one price holds in this study.

In this section, we are concerned with the link between inflation, the real exchange rate (RER) and terms of trade (TOT). We define three types of inflation in the economy: the domestic inflation $\pi_{t}$ which stems from price setting rules of domestic goods by firms, the imported inflation $\pi_{M,t}$ resulting from price setting rules of import by firms, and finally the consumer price-based inflation $\pi_{C,t}$. Taking the log-linearized form of equation (3) and then taking the first-difference yields equation (11) which is a weighted average of the two types of inflation we just mentioned.

$$\hat{\pi}_{t} = (1 - \alpha)\hat{\pi}_{H,t} + \alpha\hat{\pi}_{M,t}$$

(11)

The terms of trade is defined as follows:

$$TOT_{t} = \frac{p_{M,t}}{p_{H,t}}$$

(12)

Log-linearizing (12) around the steady-state yields the following:

$$\ell\omega_{t} = \hat{p}_{M,t} - \hat{p}_{H,t}$$

Taking the first-difference yields $\Delta\ell\omega_{t} = \hat{\pi}_{M,t} - \hat{\pi}_{H,t}$. We then substitute this in (11) to get

$$\hat{\pi}_{t} = \hat{\pi}_{H,t} + \alpha \Delta\ell\omega_{t}$$

(13)

From equation (13), it is possible to say that the difference between the total and domestic inflation rates is proportional to the terms of trade and that proportionality increases with the degree of openness of the domestic economy.

Furthermore, we define the real exchange rate $RER_{t}$ through the following relationship:
\[ RER_t = \frac{S_t P^w_t}{P_t} \tag{14} \]

Under the hypothesis of complete pass-through the price of import in domestic currency is given by \( P_{M,t} = S_t P^w_t \), which means that any idiosyncratic change in exchange rate is completely split over the domestic prices. In contrast, under incomplete pass-through—which is the case in this study—the law of one price does not hold and therefore \( P_{M,t} \neq S_t P^w_t \).

The Law of one price gap is therefore given by the ratio of the foreign price index in terms of domestic currency to domestic currency price of imports.

\[ LOPG_t = \frac{S_t P^w_t}{P_{M,t}} \tag{15} \]

Note that the law of one price holds only if \( LOPG_t = 1 \). Otherwise, the Law of one price does not hold. It is worth mentioning that through this study, the law of one price holds for exports. This is a realistic assumption since it assumes that the economies we are concerned with in this study are price takers in international markets for their exports. In contrast, importing firms are monopolistically competitive and have a small degree of pricing power in the domestic market, a novelty of Monacelli’s (2005) model (see section 2.3 for more details on that). This means that when retail firms sell imported goods to domestic consumers, they charge a mark-up over their costs, creating a wedge between the world market price of foreign goods and domestic currency price of these goods when they are sold to consumers.

Ultimately, the link between the Law of one price gap, the terms of trade and the real exchange rate is obtained by combining the log-linearized versions of (12), (13), (14) and (15) as follows:

\[ r^{\bar{\alpha}} \tau_t = \overline{\log g_t} + (1 - a) T \alpha \tau_t \tag{16} \]

Equation (16) deserves some comments. It stands that the deviation from aggregate PPP is driven by two factors are the sources. The first one is due to the heterogeneity of consumption basket between domestic goods and imported goods, an effect captured by the term \((1 - a) T \alpha \tau_t\), as long as \( a < 1 \). For \( a \to 1 \), in fact, the two aggregate consumption baskets coincide and relative price variations are not required in equilibrium. The second source of deviation from PPP is due to the deviation from the law of one price, captured by movements in \( \overline{\log g_t} \).
2.3. Production sectors

There are four types of entrepreneurs in the economy: wholesale entrepreneurs, capital producers, domestic goods retailers operating both on domestic and international markets, and imported goods retailers.

2.3.1 Wholesalers and the financial accelerator

There is a continuum of perfectly competitive wholesale firms $j \in [0,1]$ producing wholesale goods with a Cobb-Douglas-type technology of production:

$$Y_t = A_t K_t(j)^{\alpha} L_t(j)^{1-\alpha}$$  \hspace{1cm} (17)

Where $A_t$ is a technology shock following an AR(1) process:

$$log(A_t) = \zeta_A log(A_{t-1}) + \epsilon_{A,t}, \text{ where } 0 < \zeta_A < 1 \text{ is a persistence parameter and } \epsilon_{A,t} \sim i.i.d(o, \sigma_{A,t}^2)$$

$K_t$ is the capital factor and $L_t$ is the labor factor supplied by households; $0 < \alpha < 1$ is the share of the capital factor in the production function.

Following Bernanke et al. (1999), we assume that firms are credit-constrained and never accumulate enough funds to fully self-finance their capital acquisitions. This assumption is taken into account by assuming that firms have a finite expected horizon. Each survives until the next period with probability $\nu$. Accordingly, the expected horizon is given by $1/(1 - \nu)$. We assume that firms borrow only in foreign market and that their borrowing is denominated in foreign currency (Elekdag and Tchakarov, 2007). Borrowings from foreign lenders are subject to payment of a risk premium denoted by $\Phi$. If $Q_t$ and $N_t$ represent the price of capital and entrepreneur’s net worth respectively, then the entrepreneurs’ net worth is expressed in each period $t$, by the following budget constraint:

$$Q_t N_{t+1} = Q_t K_{t+1} - S_t D_{E,t+1}$$  \hspace{1cm} (18)

Where $S_t$ is the exchange rate and $D_{E,t+1}$ is the entrepreneur’s foreign debt in period $t + 1$. Equation (18) tells us that the entrepreneur’s net worth is the difference between it asset and liability. An unanticipated depreciation of the exchange rate raises the cost of capital and worsens the entrepreneur’s net worth. The framework assumes that entrepreneurs are risk neutral and choose the level of capital $K_{t+1}$ and the
associated borrowing $D_{E,t+1}$ which maximize their profits. When the optimality conditions satisfying the financial contract between the borrower and the foreign lender are reached, then the expected return on capital ($E_t R_{K,t+1}$) is equal to the marginal cost of the external fund, that is, the gross interest rate of the rest of the world $R^\omega$ adjusted for unanticipated swings in exchange rate plus country-specific risk-premium $\Psi_{D,t}$ and external finance premium $\Phi$.

$$E_t R_{K,t+1} = \Phi \left( R^\omega_{D,t} \left( \frac{S_{t+1}}{S_t} \frac{P_t}{p_{t+1}} \right) \right)$$  \hspace{1cm} (19)

The external finance premium $\Phi$ depends on the entrepreneur’s net worth $\frac{P_t N_{t+1}}{Q_t K_{t+1}}$. In general, it varies inversely with the entrepreneur’s net worth. The claim is that the greater the share of capital that the entrepreneur can either self-finance or finance with collateralized debt, the smaller the expected bankruptcy costs and, the smaller the external finance premium: $\Phi = \left( \frac{N_{t+1}}{q_t K_{t+1}} \right)^{-\gamma}$ where $\gamma$ is the elasticity of the external finance premium with respect to entrepreneurs’ net worth capital ratio, and $q_t$ is the price of capital in real terms ($q_t = \frac{Q_t}{P_t}$) and $(\Phi)' < 0; \Phi(1) = 1$.

Equation (19) provides the basis for the financial accelerator since it links movements in the borrower financial position to the marginal cost of funds and, hence, to the demand for capital.

Now we link the return to the entrepreneur’s capital $R_{K,t}$ with the marginal productivity of capital $mpc_t$. The gross return on investment per unit of capital is measured as the sum of the marginal productivity of capital arising from the production process plus non depreciated value of capital:

$$R_{K,t} q_{t-1} = mpc_t + (1 - \delta) q_t$$  \hspace{1cm} (20)

Where $\delta$ is the rate of depreciation of capital.

Finally, the relation describing the evolution of entrepreneurial net worth $N_{t+1}$ is worth mentioning. It can be expressed as a function of the value of entrepreneurial firms’ capital, net of borrowing costs carried over

\footnote{Interested readers should refer to Bernanke et al (1999) for more details on the optimization problems arising from the financial contracts between the two parties.}
from the previous period \( \nu \left[ R_{K,t} q_{t-1} K_t - R^w_{t} \Psi_{D,t} \left( \frac{S_t}{S_{t-1}} \frac{P_{t-1}}{P_t} \right) \left( \frac{N_t}{q_{t-1} K_t} \right)^{-\gamma} (q_{t-1} K_t - N_t) \right] \) plus the net worth left by firms who did not survive \((1 - \nu)\Omega_t^t\):

\[
N_{t+1} = \nu \left[ R_{K,t} q_{t-1} K_t - R^w_{t} \Psi_{D,t} \left( \frac{S_t}{S_{t-1}} \frac{P_{t-1}}{P_t} \right) \left( \frac{N_t}{q_{t-1} K_t} \right)^{-\gamma} (q_{t-1} K_t - N_t) \right] + (1 - \nu)\Omega_t 
\]  

(21)

Where \( \nu \) is the proportion of firms who survive in the economy and \( \Omega_t \) is the net worth of firms who do not survive and leave the economy each time.

Equation (21) clearly shows that the evolution of entrepreneurs’ net worth is driven by the return on investment \( R_{K,t} \) and world interest rate on borrowings, supplemented with country-specific risk premium \( (R^w_{t} \Psi_{D,t}) \). As the interest rate increases, the entrepreneur is not inclined to borrow in the foreign market, everything else being equal, and this reduces the availability of resource in the next period. The last source of fluctuations in the firms’ net worth is the variation of the exchange rate whose depreciation reduces the net worth.

### 2.3.2 Capital Producers

The activity pertaining to the role of capital producers in the economy consists of repairing depreciated capital goods and building new ones, all this being carried over in a competitive way. The production of new capital is subject to adjustment costs while the repair of old capital goods is not as in Eisner and Strotz (1963), Lucas (1967) and Gertler et al. (2006). It is also assumed that there is no possibility of substitution between old capital and new capital. The claim is that for the old capital to be productive, it should be repaired.

Both activities—old capital maintenance and production of new capital—use as input a composite investment good that is composed of domestic and foreign final goods:

\[
I_t = \left[ (1 - a) \frac{1}{\bar{\sigma}} (I_{H,t})^{\frac{\theta - 1}{\bar{\sigma}}} + (a) \frac{1}{\bar{\sigma}} (I_{M,t})^{\frac{\theta - 1}{\bar{\sigma}}} \right]^{\frac{\theta}{\theta - 1}}
\]

(22)

The associated investment price index is denoted by \( P_t \). The number of units of investment goods required to replace the depreciated capital is \( \delta K_t \) whose costs are bore by the entrepreneurs who own the capital.
stock. Therefore the amount of the investment good used for the construction of new capital goods is given by \( I_t - \delta K_t \). The adjustment costs associated with the production of new capital is given by the following quadratic form: 
\[
\psi_t \left( \frac{I_t - \delta K_t}{K_t} \right)^2 K_t,
\]
unlike in Gertler et al. (2006) where it has a linear form. Furthermore, the law of motion of capital in the economy is given by:
\[
K_{t+1} = \left[ \frac{I_t}{K_t} - \frac{\psi_t}{2} \left( \frac{I_t - \delta K_t}{K_t} \right)^2 \right] K_t + (1 - \delta) K_t
\]  
(23)

Individual capital producers choose inputs \( I_t \) and \( K_t \) to maximize expected profits from the construction of new investment goods. If \( q_t \) denotes the price of capital, then capital producers solve the following programme: 
\[
\max_{I_t} q_t I_t - \frac{\psi_t}{2} \left( \frac{I_t - \delta K_t}{K_t} \right)^2 K_t.
\]

Solving this programme yields the following optimality conditions:
\[
q_t - \psi_t \left( \frac{I_t - \delta K_t}{K_t} \right) = 1
\]  
(24)

Equation (24) is the famous Tobin Q which stems from a distortion induced by the cost of capital on price of capital. Therefore, the price of capital is variable by virtue of the adjustment cost. In the absence of investment cost (\( \psi_t = 0 \)), then \( q_t \) is identically the unity. The more the adjustment cost increases, the less the producers of capital are inclined to produce new capital. Subsequently, the price of capital increases which in turn affects negatively the entrepreneurial balance sheet in (21).

### 2.3.3 Price Setting

#### 2.3.3.1. Price setting by domestic retailers

One important feature of the model is the accommodation of the assumption of Calvo (1983) type staggered-price setting.\(^8\) We assume there is a continuum of monopolistically competitive domestic firms buying wholesale goods from producers in a competitive way and then repackage them as final goods without any cost. It is assumed that retailers of the final goods are monopolistically competitive on domestic

---

\(^8\) There are many reasons for the firm to charge a price level different from the optimal price level in the short run: menu cost, staggered prices, coordination failure, etc (Snowdon and Vane, 2005).
market whereas they are perfectly competitive on the international market. Put another way, the law of one price holds when they export, which is not the case for imports. Therefore, they sell in the foreign markets at the domestic price adjusted with the exchange rate as follows: \( P_{X,t} = \frac{P_{H,t}}{s_t} \).

In the domestic market the retailers set prices in the Calvo (1983) type price rigidity as follows: at a given point in time, a constant fraction \( (1 - \phi^H) \) of randomly selected domestic retailers set prices optimally, while the other fraction \( \phi^H \in [0, 1] \) keeps its price unchanged. Accordingly, the expected time the price of domestic goods remains unchanged is \( 1/(1 - \phi^H) \). Furthermore, we assume that those firms who can reset their prices are of two types in the economy: “forward-looking firms” and “backward-looking firms”. Forward-looking firms are those firms that reset their prices optimally by exploiting all available information at the time of making decision. The backward-looking firms unlike set their prices based on rules of thumb. They assume the information they use is “sticky” so they collect and process it with delay at the time they set their prices optimally. Basically, they use their knowledge of the historical development of price levels (which is referred to as backward-looking).

Following Gali and Gertler (1999) and more recently Smeets and Wouters (2002), backward-looking firms are assumed to reset their prices, \( P_{H,t}^f(f) \) by indexing it to the last period inflation. Therefore the parameter \( \phi^H \) becomes a natural index of price stickiness. The index of domestic prices is therefore defined as:\(^{10}\)

\[
P_{H,t}^f(f) = P_{H,t-1}(f) \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{-\phi^H}
\]

(25)

The aggregate domestic price is given by:

\[
P_{H,t} = \left\{ (1 - \phi^H) \bar{P}_{H,t}^{1-\rho} + \phi^H \left[ P_{H,t-1} \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\phi^H} \right]^{\frac{1}{1-\rho}} \right\}^{\frac{1}{1-\rho}}
\]

(26)

---

\(^9\) This is realistic as it assumes that the country has no market power when selling in the international market

\(^{10}\) This is a crude assumption since it assumes that the degree of price stickiness is the same as the fraction of past inflation indexation. However, it validates a basic rationale of Philips curve. “In the long-run Philips curve is vertical”.

---
Where $P_{H,t}(f)$ is the new price each domestic firm $f$ sets in order to maximize the present market value of its stream of expected future profits.

Log-linearizing (26) around the steady-state and taking the first difference yields the following relation for domestic inflation:

$$\pi_{H,t} = (1 - \phi^H)(\bar{P}_{H,t} - P_{H,t-1}) + (\phi^H)^2\pi_{H,t-1} \quad (27)$$

Firms re-optimize their prices and maximize their profits after setting the new price $\bar{P}_{H,t}(f)$ at time $t$ as:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} (\phi^H)^k E_t [Q_{t,t+k} Y_{t+k}(f)(\bar{P}_{H,t} - MC_{H,t+k} P_{H,t+k})]$$

Subject to the following demand function:

$$Y_{t+k} \leq \left( C_{H,t+k} + \bar{C}_{H,t+1} \right) \left[ \bar{P}_{H,t} \right]^{-\varepsilon}$$

Where $MC_{H,t+k}$ is the real marginal cost and $Q_{t,t+k} = \frac{1}{\bar{R}_{t+1}}$ is a discount factor. The first order condition of the above programme is given by:\footnote{See Chuantantikamon (2008) and Haider and Khan (2008) for more detail.}

$$\sum_{k=0}^{\infty} (\phi^H)^k E_t [Q_{t,t+k} Y_{t+k}(f)(\bar{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} NMC_{t+k})] = 0 \quad (28)$$

Where $NMC_{t+k} = MC_{H,t+k} P_{H,t+k}$ and $\frac{\varepsilon}{\varepsilon - 1}$ is considered as the marginal cost when all prices are flexible (see Gali, 2008).

Replacing $Q_{t,t+1}$ by its expression in equation (7) and log-linearizing around zero-inflation steady-state, we obtain:

$$\bar{P}_{H,t} = \bar{P}_{H,t-1} + \bar{R}_{H,t} + \sum_{k=1}^{\infty} (\beta\phi^H)^k \left[ E_t [\bar{R}_{H,t+k}] + (1 - \beta\phi^H) E_t [\bar{MC}_{t+k}] \right] \quad (29)$$
Solving equation (29) recursively and rearranging it yields the following New Keynesian Phillips Curve (NKPC):

$$\bar{P}_{H,t} - \bar{p}_{H,t-1} = \beta \phi^H E_t[\bar{p}_{H,t+1}] + \bar{p}_{H,t} + (1 - \beta \phi^H) \bar{mc}_t$$  \hspace{1cm} (30)

Furthermore, replacing equation (27) in (30), we obtain ultimately the following hybrid Phillips curve:

$$\hat{p}_{H,t} = (1 - \beta \phi^H) E_t[\hat{p}_{H,t+1}] + \phi^H \hat{p}_{H,t-1} + \frac{(1 - \beta \phi^H)(1 - \phi^H)}{\phi^H} \bar{mc}_t$$  \hspace{1cm} (31)

Where $\bar{mc}_t$ denotes log-deviation of marginal cost from its steady state value. The NKPC equation (31) implies that home country’s inflation dynamics derives from both forward looking $E_t[\hat{p}_{H,t+1}]$ and backward looking $\hat{p}_{H,t-1}$ components. The above NKPC representation is also known as a hybrid version of NKPC with forward looking and backward looking behavior. Furthermore, the equation shows that real marginal cost is also a main determinant of domestic inflation.

### 2.3.3.2. Price setting by import goods retailers

As with the domestic firms, we assume that the importing firms operate in a monopolistically competitive market. What is distinguishing here is that the law of one price gap plays an import role in determining the inflation dynamics of imported goods. Since the law of one price fails to hold, then the price index of imports in domestic currency is no longer equal to the nominal exchange rate times the foreign price index $(P_{t,M} \neq S_t P_{t,F}^M)$. Like previously shown as regard the domestic price setting of the domestic goods, domestic price of imported goods follows a Calvo-type price staggering. It implies that at a given point in time a fraction $(1 - \phi^M)$ of firms adjust their prices while the remaining $\phi^M$ cannot. Furthermore, we assume that among those firms who reset their prices some are “forward-looking” while the others are “backward-looking” firms.

The process underpinning the domestic price setting of importing firms is similar to the one defined in the case of domestic goods in equation (29). Therefore the monopolistically competitive importer’s optimal price behavior could be defined as:

$$\bar{P}_{M,t} = \bar{p}_{M,t-1} + \hat{p}_{M,t} + \sum_{k=1}^{\infty} (\beta \phi^M)^k \{ E_t[\hat{p}_{M,t+k}] + (1 - \beta \phi^M) E_t[\bar{mc}_{t+k}]\}$$  \hspace{1cm} (32)
Where \( \bar{m}c_t = \frac{S_t p^w}{p_{M,t}} = LOPG_t \) is the real marginal cost of imported goods. Solving recursively (32) and rearranging, we obtain the following new Keynesian Phillips curve relations which relates the rate of inflation in the average domestic currency price of imports to three factors: the lagged inflation rates, the expected future inflation rates and the law of one price gap.

\[
\hat{\pi}_{M,t} = (1 - \beta \phi^M) E_t \left[ \hat{\pi}_{M,t+1} \right] + \phi^M \hat{\pi}_{M,t-1} + \frac{(1-\beta\phi^M)(1-\phi^M)}{\phi^M} \log g_t 
\]

Finally, log-linearizing the consumer price index given by equation (3) around the steady-state and then taking the first difference we obtain the following log-linear form of overall inflation which is an average of domestic and imported inflations.

\[
\hat{\pi}_t = (1 - a) \hat{\pi}_{H,t} + a \hat{\pi}_{M,t} 
\]

### 2.4. Exchange rate regimes

The Taylor-type-rule (both simple and modified) of monetary policy is more and foremost the policy rule used to study the behavior of monetary authority in the DSGE literature. The modified Taylor-type-rule specifies a reaction function of nominal interest rate in response to deviations of inflation, a measure of output and exchange rate from their steady-state values. In this setting, the monetary authority enacts to stabilize the three targets: inflation, output and exchange rate.

As pointed out from the outset, the objective of this study is to make the nexus between credit-market distress and exchange rate regimes, as well as find out how some key macroeconomic variables evolve as a result of that in the context of the prospective currency union of the ECOWAS. Therefore, we consider shocks to the economy under three different scenarios: (i) a floating exchange rate regime, where the monetary authority manages the nominal interest rate according to a Taylor rule; (ii) a pure fixed exchange rate regime and (iii) target zones, where the exchange rate is allowed to fluctuate within a band between the floating exchange rate regime and the peg.

#### 2.4.1. Floating exchange rate regime

The Taylor-type reaction function of the central bank under this monetary policy regime is as follows:

\[
\hat{r}_t = \beta_0 \hat{r}_{t-1} + (1 - \beta_0) [\beta_1 \hat{\pi}_t + \beta_2 \hat{y}_t + \beta_3 \Delta \hat{s}_t] + \varepsilon_{r,t} 
\]
Where $\hat{r}_t$, $\hat{\gamma}_t$, $\hat{\gamma}_t$, and $\Delta \hat{\delta}_t$ are log-deviation of nominal interest rate, inflation, GDP and depreciation of exchange rate respectively from their steady-state values. $\beta_1$, $\beta_2$, and $\beta_3$ are weight put by monetary authorities respectively on inflation, GDP and variations of exchange rate. The lagged interest rate ($\hat{r}_{t-1}$) serves for interest rate smoothing. Finally, $\beta_0$ denotes the extent of persistence of interest rate and $\varepsilon_{r,t}$ is a random shock to interest rate with $\varepsilon_{r,t} \sim i.i.d. (0, \sigma^2_{\varepsilon_{r}})$.

### 2.4.2. Pure fixed exchange rate regime

Under this regime, the central bank keeps the nominal exchange rate pegged at a predetermined level such as $S_t = \tilde{S}$, $\forall t$. Subsequently, it sets the nominal interest rate to satisfy the uncovered interest parity condition given by equation (7) and (8).

### 2.4.3. The target zone policy

The target zone refers to the ideal range of exchange rates the monetary authorities seek to maintain within an implicit boundary. Indeed, the central bank adopts an exchange rate peg while allowing it to float around a central parity within a target band.

Following Sangaré (2013)’s method which in turn is based on Svensson (1994), the nominal exchange rate of the country is decomposed as follows: $S_t = S^c_t + S^\nu_t$, where $S^c_t$ stands for the central parity of the exchange rate and $S^\nu_t$ denote the deviations of the exchange rate from the central parity. It follows that the expected realignment of the exchange rate is given by:

$$E_t(S^c_{t+1} - S_t) = E_t(S^c_{t+1} - S^c_t) + E_t(S^\nu_{t+1} - S^\nu_t)$$  \hspace{1cm} (36)

In addition, it is assumed that the expected variations around the central parity $E_t(S^c_{t+1} - S^c_t)$ is endogenous and depends on an exogenous component $g_t$ which follows an AR (1) process:

$$E_t(S^c_{t+1} - S^c_t) = g_t + \rho_v S^\nu_t$$  \hspace{1cm} (37)

where $g_t = \rho_v g_{t-1} + \varepsilon_{g,t}$ and $\varepsilon_{g,t} \sim i.i.d. (0, \sigma^2_{\varepsilon_{g}})$

Then plugging (37) in (36) results in the following equation of exchange rate realignment:

$$E_t(S^c_{t+1} - S_t) = E_t(S^\nu_{t+1}) + g_t - (1 - \rho_v)S^\nu_t$$  \hspace{1cm} (38)
Finally, by substituting the depreciation of the nominal exchange rate \( (Δ\hat{s}_t) \) in (38) with the deviations of the exchange rate from the central parity \( (S_t^u) \), we obtain the following modified Taylor rule:

\[
\hat{r}_t = \beta_0 \hat{r}_{t-1} + (1 - \beta_0) \left[ \beta_1 \hat{r}_t + \beta_2 \hat{y}_t + \beta_3 \hat{\pi}_t \right] + \varepsilon_{r,t}
\]

(39)

2.5. The external sector

The external sector or the rest of the world is modeled in a symmetric manner relative to the domestic economy since exports from the domestic country are defined as imports of the rest of the world from that country. Therefore, similar to the optimal domestic demand for imported goods in equation (4), the optimal demand of domestically produced goods by the rest of the world is given by:

\[
C_{H,t}^\omega = a \left( \frac{P_{H,t}^\omega}{P_t^\omega} \right)^{-\theta} Y_t^\omega
\]

(40)

Where \( Y_t^\omega = C_t^\omega \) is the total demand of the rest of the world. Since the law of one price holds for exports, the price of the domestic goods in the foreign market is \( P_{H,t}^\omega = \frac{P_{H,t}}{S_t} \).

Subsequently, rearranging equation (40) yields the following expression of foreign demand as a function of the real exchange rate:

\[
C_{H,t}^\omega = a \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left( \frac{P_t}{S_t P_t^\omega} \right)^{-\theta} Y_t^\omega = a \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left( \frac{1}{RER_t} \right)^{-\theta} Y_t^\omega
\]

(41)

Finally the following foreign variables are modeled as exogenous in the model. Following the literature, we assume that they follow first order autoregressive processes:

\[
\hat{\pi}_t^\omega = \zeta_{\pi \omega} \hat{\pi}_{t-1}^\omega + \varepsilon_{\pi \omega,t}
\]

(42)

\[
\hat{y}_t^\omega = \zeta_{y \omega} \hat{y}_{t-1}^\omega + \varepsilon_{y \omega,t}
\]

(43)

\[
\hat{\rho}_t^\omega = \zeta_{\rho \omega} \hat{\rho}_{t-1}^\omega + \varepsilon_{\rho \omega,t}
\]

(44)

Where \( \hat{\pi}_t^\omega \), \( \hat{y}_t^\omega \) and \( \hat{\rho}_t^\omega \) represent the log-deviation of foreign interest rate, foreign GDP and foreign inflation respectively from their steady-state and \( \varepsilon_{i,t} \) is an i.i.d normal error term with zero-mean and standard deviation of \( \sigma_i \), where \( i = r \omega, y \omega \) and \( \pi \omega \).
2.6. Equilibrium

The system of the model consists of the optimality conditions (see equation 1 till 34), the government budget constraint, the monetary policy regimes, market clearing conditions, the balance of payment, and processes of the exogenous shocks.

For simplicity, it is assumed that the sole role of the government in the economy consists of receiving lump-sum tax ($T_t$) from households and then transferring it ($T_t$) to the same households.\(^{12}\) Therefore, the government budget constraint is simply given by:

$$T_t = \tau_t$$

The equilibrium conditions in each market are as follows:

The financial market: $B_t = 0$

The labor market: $L_t = \int_0^1 L_t(f)df$

The domestic goods market: $Y_t = C_{H,t} + I_{H,t} + X_t$, where $X_t$ stands for total exports of the country.

Using demand functions defined in (4) and (40), the aggregate demand is given by:

$$Y_t = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} (C_t + I_t) + \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} \left[ \alpha \left( \frac{1}{RER_t} \right)^{-\theta} Y_t^{w_0} \right]$$

(45)

Net foreign asset position (balance of payments) of the country is given by:

$$S_t D_t = S_t R_t^{w_0} D_{t-1} \Psi_{D,t-1} + X_t - M_t$$

(46),

where $M_t$ stands for total imports of the country.

The dynamic of the net foreign position of the country therefore depends on the current account balance as well as the interest payments on the previous period debt.

We then express equation (46) relative to GDP:

\(^{12}\) See Medina and Soto, 2007; Fernández-Villaverde and Ohanian, 2009 for a model with a fully fledged fiscal sector.
2.7. Stochastic Process

The economy is subject to five orthogonal AR (1) stochastic shocks representing log-linear deviation from the steady-state: a country risk premium shock ($\tilde{z}_t$) which is a shock on country borrowing premium; a domestic productivity shock ($\tilde{A}_t^Y$); a foreign interest rate shock ($\tilde{r}_t^\omega$) considered as a shock in foreign financial conditions such as increasing risk premium; a foreign demand shock ($\tilde{g}_t^\omega$); and a foreign inflation shock ($\tilde{\pi}_t^\omega$).

3. Calibration and Estimation Strategies

The empirical literature offers numerous strategies for the determination of the parameters of New Keynesian DSGE models ranging from pure calibration to econometric estimation or a mix of both. This study builds on the latter strategy and is aimed to partially calibrate and estimate the log-linearized version of the model laid out in appendix. The estimation strategy uses actual data for the founding members of the WAMZ (The Gambia, Ghana, Guinea, Nigeria and Sierra Leone) and WAEMU.

3.1. Calibration

The tradition in calibration consists to borrow the parameters’ values from the literature on economies of similar structure. It turns out to be the easiest way towards making the theoretical model mimic and reproduce stylized facts about economies (DeJong and Dave, 2007). The calibrated model then serves as reference when assessing the dynamics of some key macroeconomic variables following a random shock hitting the economy. In this paper, we borrow most of the parameters’ values from the literature on economies of Sub-Saharan Africa countries. Parameters for which there is no literature available will be assigned values using values for developed countries as reference. The model is solved numerically using DYNARE toolbox and generates impulse response functions to the shocks mentioned in section 2.7. The complete list of the calibrated parameters, their values and their sources are in table 1.

---

For sake of economy of space, we report only the results from country risk premium shock. The other results are available upon request.

DYNARE is a user friendly MATLAB toolkit which solves, estimates and simulates DSGE models as well as other models. See http://www.dyanre.org/ more informations.
Table 1: Baseline parameters calibration*

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>values</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>inverse of intertemporal elasticity substitution for consumption</td>
<td>1/0.34</td>
<td>Ogaki et al. (1996)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>inverse of Frisch elasticity of labour supply</td>
<td>10.0</td>
<td>Berg et al. (2012)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>agents’ discount factor</td>
<td>0.91</td>
<td>Berg et al. (2012)</td>
</tr>
<tr>
<td>$\psi_D$</td>
<td>elasticity of country risk premium on FX borrowing</td>
<td>0.0007</td>
<td>Schmitt-Grohe and Uribe (2003)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>production parameter for private capital</td>
<td>0.40</td>
<td>Araujo et al. (2013)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>probability of firms surviving in the economy</td>
<td>0.9728</td>
<td>Bernanke et al (1999)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>elasticity of firms’ risk premium on FX borrowing</td>
<td>1.00</td>
<td>Elekdag and Tchakarov (2007)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate for private capital</td>
<td>0.1</td>
<td>Berg et al. (2012)</td>
</tr>
<tr>
<td>$\psi_I$</td>
<td>capital adjustment cost</td>
<td>0.25</td>
<td>Araujo et al. (2013)</td>
</tr>
<tr>
<td>$K/N$</td>
<td>capital/firms’ net worth in initial state</td>
<td>3.00</td>
<td>Devereux et al. (2006)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>elasticity of substitution between different varieties of goods</td>
<td>0.44</td>
<td>Berg et al. (2012)</td>
</tr>
</tbody>
</table>

*Parameters' values for individual countries are reported in appendix

3.2. Estimation Strategies

The structural parameters characterizing the economy in the theoretical model described above are estimated. These are basically the parameters characterizing the monetary policy rule; elasticities of substitution between domestic and foreign goods, parameters related to the Calvo-type price rigidity, parameters pertaining to the persistence of stochastic shocks, and standard errors related to the shocks.

Many estimation methods of the DSGE models have been put forward in the literature. We distinguish among them the maximum likelihood method, the generalized moment method and Bayesian method. In this study we use Bayesian estimation techniques for the model estimation. We follow the same presentation form of Beidas-Strom (2011) for our model estimation. The complete log-linearized version of the model previously described is presented in appendix and can be written in the form of linear system with rational expectation as follows:

$$\Omega(\theta)z_t = \Omega_1(\theta)z_{t-1} + \Omega_2(\theta)e_t + \Omega_3(\theta)\xi_t$$  \hspace{1cm} (48)

\hspace{1cm} 15 See Ruge-Murcia (2007) for a comparative study on these methods.
Where

\[ z_t = \{ \bar{y}_t, \tilde{c}_t, \bar{q}_t, \bar{l}_t, \bar{w}_t, \bar{m}_{pc} \bar{c}_t, \bar{\hat{n}}_{H,t}, \bar{\hat{n}}_{M,t}, \bar{\hat{n}}_{R,t}, \Delta \ln \bar{p}_t, \bar{\hat{k}}_{t+1}, \bar{\hat{n}}_{t+1}, \bar{\hat{d}}_t \} \]

is a vector containing the model’s endogenous variables expressed as log-deviations from their steady-state values, and

\[ \varepsilon_t = \{ \varepsilon_{A,t}, \varepsilon_{z,t}, \varepsilon_{y,t}, \varepsilon_{\varepsilon_{\omega,t}}, \varepsilon_{\varepsilon_{\omega,t}} \} \]

is a vector of innovations to stochastic shocks and coefficients matrices \( \Omega_t \) are non-linear functions of the structural parameters contained in \( \vartheta \). The solution to the system can be written as follows:

\[ z_t = \Omega_2(\vartheta)z_{t-1} + \Omega_3(\vartheta)\varepsilon_t \quad (49) \]

Relations (48) and (49) stems from measurement equations linking observable variables used in the estimation with endogenous and exogenous variables. We can express them through a single equation as follows:

\[ y_t^T = Hz_t \quad (50) \]

Where \( y_t^T = \{ \bar{y}_t, \tilde{c}_t, \bar{\hat{n}}_{H,t}, \Delta \hat{s}_{t}, \hat{r}_t \} \) is a vector of observable variables used in the estimation and \( H \) is a deterministic matrix. Equations (48), (49) and (50) form the state-space representation of the model, the likelihood of which can be evaluated using Kalman filter provided the white innovations are normally distributed. In practice, the Bayesian approach first place a prior distribution with density \( P(\vartheta) \) on structural parameters \( \vartheta \). It then uses the data, \( y_t^T \), to update the prior distribution through the likelihood function, \( L(\vartheta|y_t^T) \). From this updating process we obtain the posterior distribution of \( \vartheta \) according to Bayes’ theorem:

\[ P(\vartheta|y_t^T) = \frac{L(\vartheta|y_t^T)P(\vartheta)}{\int L(\vartheta|y_t^T)P(\vartheta)d\vartheta} \quad (51) \]

Posterior distributions are generated using Markov-Chain-Monte-Carlo (MCMC) simulation methodology which is briefly discussed in Lubik and Schorfheide (2005); Gelman et al. (2006) and Koopman et al. (2007). Finally, the simulation techniques use the random walk Metropolis-Hastings (MH) algorithm.

The parameter vector to be estimated in this study is:
\[ \theta = \{ h, \Theta, \phi^H, \phi^M, \zeta, \zeta_A, \zeta_{\pi}, \zeta_{\omega}, \sigma_{\epsilon}, \sigma_{\epsilon_{\pi}}, \sigma_{\epsilon_{\omega}}, \beta_0, \beta_1, \beta_2, \beta_3, \sigma, \sigma_{\pi}, \sigma_{\omega}, \} \]

### 3.2.1. Data

For the five countries of the WAMZ plus the WAEMU sub-region under study, we rely on data drawn from the *World Economic Outlook (2010)* and the IMF’s *International Financial Statistics (2012)*. The data are of annual frequency spanning from 1980 to 2010 and the selected observable variables include real GDP; consumption; overall domestic inflation; real exchange rate; and nominal interest rate. Since the model variables are expressed in terms of log-deviations from their steady-state values, we pre-process them. Basically, this consists of seasonally adjusting the variables using filtering techniques. The most commonly used approach is the Hodrick-Prescott (HP) filter we build on in this paper. In the case of real GDP, we detrend the series in order to work with stationary series. Consumer price inflation is used as a measure of the overall domestic inflation as well as to construct real exchange rate.

### 3.2.2. Prior distribution

Priors’ distributions (mean and standard deviation) are gleaned from personal belief about parameter value and economic theory (Schorfiede, 2000). In practice, priors are chosen on the base of theoretical restrictions on the parameter values (non-negativity or confidence interval) given in the existing literature. *Beta* distribution is chosen for parameters with values constrained in interval \([0, 1]\). *Gamma* and *normal* distributions pertain to parameters values that are non-negative while *inverse gamma* distribution is used for the distribution of standard deviation of shocks.

In line with the empirical onslaught pertaining to DSGE models with application in Sub-Saharan African economies (Peiris and Saxegaard, 2010; Dagher et al. 2010; Berg et al. 2010; Senbeta 2011; Diop et Fall. 2011; Berg et al. 2012; Araujo et al. 2013), we draw the prior distribution for each parameter contained in \( \theta \), its mean and standard deviation. For the degree of habit persistence in consumption, \( h \), we assume a truncated normal distribution with mean 0.70 and standard deviation equal to 0.15. Similar to Diop et Fall. (2011), the parameters measuring the degree of Calvo price stickiness \( (\phi^H) \) and \( (\phi^M) \) are assumed to have the same mean 0.50 and standard deviation 0.15. As regards the priors in the coefficients of the monetary policy, we place a relatively high mean on inflation coefficient \( (\beta_1) \) with mean 1.50 and standard deviation 0.25 and identically low coefficient mean value equal to 0.70 and standard deviation 0.10 for output growth coefficient \( (\beta_2) \) and exchange rate coefficient \( (\beta_3) \). The interest rate smoothing coefficient
(β₀) is assumed to follow a gamma distribution with mean set to 0.75 and standard deviation 0.15. The elasticity of substitution between foreign and domestic goods, θ, follows an inverse gamma distribution with mean 1.50 and standard deviation 0.75. Finally the AR (1) parameters (persistence coefficients) of the stochastic shocks ζₐ, ζₕ, ζₚ have gamma distribution with the same mean set at 0.50 and standard deviation at 0.20. The mean of the world interest rate smoothing parameter, ζₚ, and country risk premium ζₚ are identically set to 0.46 as is the value of the standard deviation set at 0.15, as in Devereux et al. (2006). The estimation results and impulse response functions of country risk premium shocks for all the five countries as well as for the WAEMU sub-region are given in figure A1-6 and figure B1-6 in appendix.

### 3.3. Estimation Results

In this section, we outline the estimation results of the model. First, we lay out the parameters estimates and then we discuss the impulse response functions from a risk premium shock on the dynamic of some key macroeconomic variables.

#### 3.3.1. Parameter Estimates

The combination of the suitable priors with the likelihood functions allows computing the posterior mean and constructing the posterior distribution with the Metropolis-Hastings algorithm. Figure A1-6 displays the prior distributions of the parameters, along with their posterior distributions. It turns out that the estimation results of the structural parameters fall within plausible ranges when considering the fixed exchange rate regime. Therefore, we use these parameters values to carry out the counterfactual analysis under the other monetary regimes. Furthermore, the results show that the posterior and prior mean and of most of the parameters are different from each other, which is a reflection of the data used in the updating of the priors.

The parameter of habit formation in consumption, h, is lower than its prior mean of 0.7 for all the set of countries in this study, meaning that the degree of consumption persistence in these countries is quite low as compared with developed economies (see for instance Lubik and Schorfheide (2005). The parameter estimates of the elasticity of substitution between home and foreign goods in the consumption basket of domestic households, θ, is higher than its prior mean value of 1.5 for all countries, and even higher than 2 for Guinea and 3 for Sierra Leone and WAEMU. It should be noted that a high value of this parameter

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16 The estimation results of the parameters from the target zone and flexible exchange rate regimes are available upon request.
points to a high degree of adjustment of consumers from these countries in response to changes in relative prices of domestic goods and imported goods.

The posterior estimates of Calvo price stickiness provides insights about the frequencies of price changes, through the probability of firms who do not reset their prices in a given year. The estimated posterior mean value of the Calvo probability is lower than the prior mean of 0.5 for home goods prices ($\phi^H$) and for foreign goods prices ($\phi^M$) for most of the countries except for Ghana. Indeed, unlike the other countries, the probability of changing foreign goods prices in Ghana is 0.7. Lower values of ($\phi^H$) and ($\phi^M$) show that domestic goods prices and foreign goods prices respectively are re-optimized frequently in a given year. The more the firms reset their prices in a given year, the more inflation is subdued and inversely when price setting is staggered. Therefore, the lower posterior mean (<0.5) of the probability of not resetting prices in all countries/region (except Ghana) brings into the fore that inflation is subdued in this set of countries excluding Ghana\(^{17}\). Since the expected time a price is reset is $1/(1 - \phi^i)$, with $i = H, M$, then the average duration retailers of both home goods and foreign goods set their prices is less than 2 years for most of the countries while prices of import goods are sticky over more than 2 years for Ghana. These results are in line with findings from Diop et Fall (2011) in the case of all the ECOWAS countries.

The posterior estimates of the policy rule coefficients, $\beta_0$, $\beta_1$, $\beta_2$, and $\beta_3$ provide plausible reaction function of the future Central Bank of the currency union to inflation’s deviation from its implicit target, output growth’s deviation from its potential, and exchange volatility. First, the degree of interest rate persistence ($\beta_0$) falls below the prior mean of 0.75 for all the countries. In particular, its mean value for Nigeria, Ghana and The Gambia is quite large and estimated at 0.44, 0.57, and 0.66 respectively which are close to Diop et Fall (2011)’s estimates. Second, the response of the interest rate to inflation’s deviation from its target ($\beta_1$) is estimated to be higher than the higher than the prior mean value of 1.5 for all the countries.\(^{18}\) Likewise, the output gap coefficient ($\beta_2$) is above its prior mean of 0.70 for all the countries. This finding shows that Central Banks in these countries overreact to inflation and output.\(^{19}\) The rational for

\(^{17}\)Ghana adopted inflation-targeting policy since 2007 which is deemed to keep inflation within a target band. Indeed, the country experienced many double digit-inflations, with inflation reaching 20-percent levels in 2004 to 10.7 percent by end-2010, above the mid-point target, according to IMF staff report.

\(^{18}\) Diop et Fall (2011)’s estimate of these parameters points to similar finding for Ghana, as can be expected given inflation targeting.

\(^{19}\)This is in line with finding in Siri (2009)’s estimates of the Central banks reaction function via the canonical and modified Taylor rule. It suggests that the Central Bank of the WAEMU’s reaction is higher for both inflation and growth, while Central Bank of Ghana and of Nigeria react genuinely to inflation and weakly to output gap.
the Central Banks overreaction to growth is to cope with demand side shocks in these countries. Third, the estimated coefficient of the response of interest rate to volatility in exchange rate ($\beta_3$) is above its prior mean of 0.70 for all the five countries, except for The Gambia for which it is slightly below 0.70. This is in line with Diop et Fall (2011)’s estimates.

### 3.1.2. Dynamics of the Model: Impulse Response Analysis

We now explore the model’s key features through the impulse response of endogenous variables to the financial accelerator shock.\(^20\) We perform this through a negative shock to country risk premium which in turn is captured through a 1 standard deviation decrease of innovation in the initial period. Since the ultimate goal in this study is to gauge the implication of external financial constraints on monetary policy transmission of the future Central Bank of the ECOWAS, we carry out the shock for each of the above-mentioned monetary regimes;\(^21\) fixed exchange rate regime and pure floating exchange rate regime. The results for the simulation with the posterior mean parameters are displayed in figure B1-6. In each panel, the solid line reflects the dynamic of the macro variables when the shock hits the economy under fixed exchange rate whereas the dotted red line reflects the model economy under floating exchange rate regime.

The figures provide a clear perception of the response of inflation, output, investment, real interest rate, real exchange rate and terms of trade and entrepreneurs’ net worth to a negative country risk premium shock (or a positive foreign shock) for the five countries along with the WAEMU area. As we have set up in the previous sections, the external finance premium depends inversely on borrower’s net worth.\(^22\) Furthermore, to the extent that borrowers’ net worth is procyclical,\(^23\) the external finance premium will be countercyclical (Bernanke et al. 1999). Therefore, any contemporaneous and negative country risk premium shock reduces the external borrowing cost while subsequently enhancing the external borrowing and lowering domestic

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20 Results from the other shocks mentioned above are not reported here but are available upon request.
21 Since the target zone policy is an intermediate exchange rate regime varying within a band with the fixed exchange rate regime and floating exchange rate regime being its lower and upper bound respectively, we do not explicitly implement the experiment under this regime; rather we provide a perception of its effects drawing on the two mentioned regimes.
22 This is so because when borrowers have little wealth to contribute to capital financing the potential divergence of interests between the borrower and the suppliers of external funds is greater, implying increased agency costs; in equilibrium, lenders must be compensated for higher agency costs by a larger premium (Bernanke et al. 1999).
23 This may arise because of the procyclicality of profits.
nominal and real interest rates.\footnote{This happens since there is a decrease in foreign interest rate and therefore a shift away from domestic borrowing.} This in turn leads to an increase in investment. The financial accelerator mechanism propagates and amplifies the rise in investment through entrepreneurs’ balance sheet effect. Indeed, an exogenous decrease in the country borrowing premium introduces a wedge between domestic and foreign interest rates in the uncovered interest rates parity (see equations 7 and 8). Formally, this is equivalent to a fall in foreign interest rate; nominal and real exchange rates decrease as a result of that (nominal and real appreciation).\footnote{Adolfson et al. (2008) noted that the uncovered interest rate parity (UIP) condition is a key equation in open economy DSGE models. It shows the difference between domestic and foreign nominal interest rates equals the expected future change in the nominal exchange rate. The UIP condition is a key equation in open economy models not only for the exchange rate but also for many macroeconomic variables, since there is a lot of internal propagation of exchange rate movements working through fluctuating relative prices. There is, however, strong empirical evidence against the standard UIP condition, see for instance, e.g., Eichenbaum and Evans (1995); Faust and Rogers (2003). Moreover, a DSGE model with a standard UIP condition cannot account for the so-called ‘forward premium puzzle’ recorded in the data, i.e. that a currency whose interest rate is high tends to appreciate which implies that the risk premium must be negatively correlated with the expected exchange rate depreciation see, e.g., Fama, (1984); Froot and Frankel (1989).} Because liabilities are “dollarized”, the subsequent real appreciation has beneficial effects on entrepreneurial net worth, which in turn insulate investment from falling due to financial frictions. This phenomenon which links the entrepreneurs’ balance sheet to investment, and hence to output illustrates how external financial conditions affect the economies considered in this study. However, the real appreciation has damaging effect on external position of the economy. Therefore, the final effect of the shock on economic activity is pinned down once the offsetting effect from deteriorating current account is taken into account. Nonetheless, our main results seem to suggest that only Ghana is responsive of the balance sheet effect previously described as illustrated by an increase in net worth as well as in investment as expected. For the other countries results of the balance sheet effect are counterintuitive since they go in the other way around, as shown in figure B1-6.

The impulse response of output to the positive foreign shock and the dynamic of the latter show that each individual country is affected differently, depending on the exchange rate policy considered. In fact, as pointed out, there is an offsetting effect, in that with foreign indexed-debt, the appreciation of the exchange rate reduces the foreign demand for home country goods, while creating a detrimental effect on net foreign position of each economy. In this instance, we find that there is a drop in output of all the individual countries which remains smaller under flexible exchange rate than under fixed rates. Put differently, the impact of the exchange rate on the balance sheet under flexible exchange rate is less damaging than the appreciation of asset prices under fixed rates.
For all individual countries, the impulse responses of variables (especially firms’ net worth, investment and output) when the domestic economy is affected by a negative shock on country risk premium, highlight that a flexible exchange rate remains dominant, followed by fixed rates. For example, a 1 standard deviation shock on the risk premium of Ghana generates 6 percent and 0.2 percent increases in investment in period 0 respectively under pure floating exchange rate and fixed exchange rate policy. The same shock leads to 0.18 percent and 0.27 percent decreases in output respectively under flexible exchange rate and fixed exchange rate policy. These findings are consistent with the conventional wisdom according to which flexible exchange rates are better absorber of real foreign shock than are fixed exchange rates (see Flood and Marion (1982) and Aizenman and Frenkel (1985), among many others).

The intuition is that, under fixed exchange rates \( \Delta \hat{s}_t = 0 \), real appreciations (depreciations) is only accomplished through inflation (deflation). A real appreciation, which is the case in this study, pushes up investment and output through entrepreneurial balance sheets effect. For most of the sample countries, investment is above its steady state value in period 0 following the shock, while decreasing in subsequent periods; output is below its steady state value while increasing in subsequent periods. In addition, the direction of the response of the risk premium to a real appreciation depends on the size of the elasticity of the borrowing premium with respect to the total indebtedness \( \psi_D \). Nonetheless, the value of the elasticity is identically set for the two exchange rate regimes.

4. Conclusion

Using a model of a small open economy Dynamic Stochastic General Equilibrium (DSGE) which features real and nominal rigidity, habit formation in consumer’s utility function, backward-looking and forward-looking firms, operating costs in firm’s capital utilization and imperfect capital mobility, this paper evaluates the performance of two exchange rate regimes for the five founding member of the WAMZ under a foreign shock namely a country risk premium shock. The model embeds the financial accelerator mechanism through which the terms of access to credit in international credit market and hence of demand for capital are linked with the state of borrower balance sheets. It also incorporates the phenomena of incomplete pass-through and foreign currency debt mechanism. Some parameters of the model have been calibrated while the remaining parameters have been estimated using the Bayesian simulation approach, which combines prior information drawn from the literature and from historical data covering the period 1980 to
2010. The estimates of the key structural parameters of the model fall within plausible ranges. To try to pin down how the economy responds to foreign shock and how the choice of an exchange rate regime influences that response for the ECOWAS countries, we simulate the model by modifying different policy parameters and compare the results under two policy rules: fixed exchange rate and floating exchange rate.

The main results can be summarized as follows. First, a negative country risk premium shock is equivalent to a fall in borrower risk premium through a fall in foreign interest rate. The real exchange rate decreases through the uncovered interest rate parity conditions. Since the entrepreneurial liabilities are foreign currency denominated, the real exchange rate appreciation tends to be reckless with regard to entrepreneurial net worth as well as demand for capital. Therefore, the shock is easy-going for investment as result of the firms’ balance sheet effect. Results show that among the WAMZ countries along with the WAEMU area, only Ghana is responsive of the balance sheet effect as illustrated by an increase in net worth as well as in investment as expected. For the other countries, results of the balance sheet effect are counterintuitive since they go in the other way around. Second, there is an offsetting effect since real appreciation makes export goods more expensive relative to import goods with detrimental effect on current account. Third, exchange rate-policy shocks suggest the superiority of the insulating role of a flexible exchange rate regime over that of a peg. Indeed, we find that the offsetting effect seems to dominate since there is a drop in output of all the countries. Nonetheless, the contraction in economic activity remains smaller under flexible exchange rate than under fixed rates.

Finally, results should be viewed from the angle of the model assumptions. We concede that after relaxing some of the assumptions and incorporating some others, this model could be more robust for policy decision making.
References (incomplete)


Dufrenot G. (2009a), « Credit policy stress in the West African Economic and Monetary Union ». The Developing Economies.


Appendix

A1. Log-linearized Version of the Model

(a) Demand

\[ \ddot{y}_t = (1-a) \left( \frac{c}{y} \ddot{e}_t + \frac{i}{y} \ddot{i}_t + \frac{g}{y} \ddot{g}_t \right) + a \ddot{y}_t + \theta a \left( \frac{2-a}{1-a} \right) \ddot{r} \ddot{e}_t - \frac{\theta a}{1-a} \ddot{\log} g_t \]

\[ C_t = \frac{h}{1+h} C_{t-1} + \frac{1}{1+h} E_t C_{t+1} - \frac{1-h}{\sigma(1+h)} (r_t - E_t \pi_{t+1}) \]

\[ E_t (\ddot{r}_{K,t+1}) = \ddot{r}_t - E_t \ddot{r}_{t+1} - \gamma (\ddot{q}_{t+1} - \ddot{q}_t - \ddot{k}_{t+1}) \]

\[ \ddot{r}_{K,t+1} = \left( 1 - \frac{(1-\delta) \ddot{r}}{\ddot{r}_K} \right) \ddot{m}_p c_t + \left( \frac{1-\delta}{\ddot{r}_K} \right) \ddot{q}_t - \ddot{q}_{t-1} \]

\[ \ddot{q}_t = \psi_t (\ddot{i}_t - \ddot{k}_t) \]

(b) Supply

\[ \ddot{y}_t = \ddot{A}_t + a \ddot{k}_t + (1-a) \ddot{l}_t \]

\[ w_t = \eta L_t + \frac{\sigma}{1-h} (C_t - h C_{t-1}) \]

\[ \ddot{w}_t = \ddot{y}_t + m \ddot{c}_t - \ddot{t}_t - \frac{a}{1-a} (r \ddot{e}_t - \ddot{\log} g_t) \]

\[ \ddot{m}_p c_t = \ddot{y}_t + m \ddot{c}_t - \ddot{k}_t - \frac{a}{1-a} (r \ddot{e}_t - \ddot{\log} g_t) \]

\[ \ddot{h}_t = (1-a) \ddot{h}_{H,t} + a \ddot{h}_{M,t} \]

\[ \ddot{h}_{H,t} = \left( 1 - \beta \phi^H \right) E_t \ddot{r}_{H,t+1} + \phi^H \ddot{r}_{H,t-1} + \frac{(1-\beta \phi^H)(1-\phi^H)}{\phi^H} \ddot{m}_c_t \]

\[ \ddot{h}_{M,t} = \left( 1 - \beta \phi^M \right) E_t \ddot{r}_{M,t+1} + \phi^M \ddot{r}_{M,t-1} + \frac{(1-\beta \phi^M)(1-\phi^M)}{\phi^M} \ddot{\log} g_t \]

\[ r e t = \ddot{\log} g_t + (1-a) \ddot{t} \ddot{c}_t \]

\[ \Delta \ddot{\log} g_t = \Delta S_t + \ddot{r}_t - \ddot{r}_{M,t} \]

\[ \Delta \ddot{c}_t = \ddot{h}_{M,t} - \ddot{h}_{H,t} \]
(c) Evolution of State Variables

\[ \hat{k}_{t+1} = \delta \hat{k}_t + (1 - \delta) \hat{k}_t \]

\[ \hat{n}_{t+1} = v \hat{r}_K \left[ \left( \frac{k}{n} \right) \hat{r}_{K,t} + \left( 1 - \frac{k}{n} \right) (\hat{r}_{t-1} - E_t \hat{n}_t) + \gamma \left( 1 - \frac{k}{n} \right) (\hat{a}_{t-1} + \hat{k}_t) + (1 + \gamma \left( \frac{k}{n} - 1 \right) \hat{n}_t \right] \]

\[ \hat{\alpha}_t = \frac{1}{\beta} \hat{\alpha}_{t-1} + \hat{\gamma}_t - \frac{c}{y} \hat{\epsilon}_t - \frac{i}{y} \hat{\epsilon}_t - \left( \frac{\alpha}{1 - \alpha} \right) (r \hat{\epsilon}_t - \log \epsilon) \]

\[ \hat{r}_t - E_t \hat{n}_{t+1} = \hat{r}_t^{\omega} - E_t \hat{\alpha}_{t+1}^{\omega} - \psi_D \hat{\alpha}_t + \hat{z}_t + E_t r \hat{\epsilon}_{t+1} - r \hat{\epsilon}_t \]

\[ \Delta \hat{\epsilon}_t = \Delta r \hat{\epsilon}_t - \hat{r}_t^{\omega} + \hat{n}_t \]

(d) Monetary Policy Rule

\[ \hat{r}_t = \beta_0 \hat{r}_{t-1} + (1 - \beta_0) (\beta_1 \hat{r}_t + \beta_2 \hat{y}_t + \beta_3 \Delta \hat{\epsilon}_t) + \epsilon_{r,t} \]

(e) Foreign Variables

\[ \hat{r}_t^{\omega} = \zeta_{r \omega} \hat{r}_{t-1}^{\omega} + \epsilon_{r \omega,t} \]

\[ \hat{\gamma}_t^{\omega} = \zeta_{\gamma \omega} \hat{\gamma}_{t-1}^{\omega} + \epsilon_{\gamma \omega,t} \]

\[ \hat{\alpha}_t^{\omega} = \zeta_{\alpha \omega} \hat{\alpha}_{t-1}^{\omega} + \epsilon_{\alpha \omega,t} \]

(f) AR(1) Process of Stochastic Shocks

\[ \hat{A}_t^{\gamma} = \zeta_A \hat{A}_{t-1}^{\gamma} + \epsilon_{A,t} \]

\[ \hat{z}_t = \zeta_{z} \hat{z}_{t-1} + \epsilon_{z,t} \]

\[ \hat{\theta}_t = \rho_y \hat{\theta}_{t-1} + \epsilon_{g,t} \]

A2. Estimated Parameters and Impulse Response Functions
Figure A1: Ghana: Estimated parameters
Figure B1: Ghana: Impulse response to a country risk premium shock—$$\varepsilon_{z,t}$$
Figure A2: Gambia: Estimated parameters
Figure B2: Gambia: Impulse response to a country risk premium shock—$\epsilon_{z,t}$
Figure A3: Guinea: Estimated parameters
Figure B3: Guinea: Impulse response to a country risk premium shock—$\varepsilon_{z,t}$
Figure A4: WAEMU: Estimated parameters

- phim
- betha0
- betha1

- betha2
- betha3
- rhoyw
- rhow
- rhopiw
- rhoz

- SE er
- SE eyw
- SE erw

- SE epiw
- SE ez
- SE eay
- h
tetha
- phih

- rhoay
- rhog
Figure B4: WAEMU: Impulse response to a country risk premium shock—$\epsilon_{x,t}$
Figure A5: Nigeria: Estimated parameters

- phim
- beta0
- beta1
- beta2
- beta3
- rhoyw
- rhoyw
- rhoiw
- rhoz
- SE_er
- SE_eyw
- SE_erw
- SE_eplw
- SE_ez
- SE_eay
- h
- phih
- rhoay
- rhog
Figure B5: Nigeria: Impulse response to a country risk premium shock—$\varepsilon_{z,t}$
Figure A6: Sierra Leone: Estimated parameters
Figure B6: Sierra Leone: Impulse response to a country risk premium shock—$\varepsilon_{z,t}$